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Environmental and Resource Economics
Leseprobe

Fakultät für
**Wirtschafts-
wissenschaft**

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Environmental and Resource Economics

Part 1:
Environmental Economics

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1 Background information on climate change

Last decade was Earth's hottest on record as climate crisis accelerates[1]

- *2019 was second or third hottest year ever recorded*
- *Average global temperature up 0.39 °C in 10 years*

The past decade was the hottest ever recorded globally, with 2019 either the second or third warmest year on record, as the climate crisis accelerated temperatures upwards worldwide, scientists have confirmed.

Every decade since 1980 has been warmer than the preceding decade, with the period between 2010 and 2019 the hottest yet since worldwide temperature records began in the 19th century. The increase in average global temperature is rapidly gathering pace, with the last decade up to 0.39C warmer than the long-term average, compared with a 0.07C average increase per decade stretching back to 1880.

The past six years, 2014 to 2019, have been the warmest since global records began, a period that has included enormous heatwaves in the US, Europe and India, freakishly hot temperatures in the Arctic, and deadly wildfires from Australia to California to Greece.

Last year was either the second hottest year ever recorded, according to Nasa and the National Oceanic and Atmospheric Administration, or the third hottest year, as recorded by the UK Met Office. Overall, the world has heated up by about 1C on average since the pre-industrial era.

"As this latest assessment comprehensively confirms, we have just witnessed the warmest decade on record," said Michael Mann, a climate scientist at Penn State University. "As other recent reports confirm, we must act dramatically over this next decade, bringing emissions down by a factor of two, if we are to limit warming below catastrophic levels of 1.5C that will commit us to ever-more dangerous climate change impacts.

"This is something every American should think about as they vote in the upcoming presidential election."

[1] The following paragraphs are all from https://www.theguardian.com/us-news/2020/aug/12/hottest-decade-climate-crisis-2019 [visited August 14 2020].

2 Neoclassical welfare economics

The starting point of most economic analysis is the 'neoclassical general equilibrium theory'. This is the benchmark against which economists usually compare any types of inefficiency or market failure, e.g. related to imperfect competition or monopoly, incomplete information, or public good problems and externalities (e.g. related to environmental issues). The 'nice' thing about perfect competition in the neoclassical framework is, that it leads to Pareto efficiency, hence, a situation where nobody can be made better off unless somebody else is made worse off ("First Fundamental Theorem of Welfare Economics"). This is also a situation that society may want to achieve, because if somebody can be made better off without making someone else worse off, the social "well-being" can unambiguously be improved, hence, the original allocation was clearly not the best possible outcome.

Unfortunately, a Pareto efficient allocation can be very 'unfair', e.g. if some individuals are particularly poor and others rich. Therefore, the problem is that Pareto efficiency alone says nothing about equity. To judge which of the many (in fact infinitely many) Pareto efficient allocations is 'optimal' (the most desirable one from a social point of view), society (or philosophers) must come up with some definition of 'social welfare'. Whatever this definition might be, another nice thing about the neoclassical theory is, that it predicts that the resulting socially optimal allocation (one specific allocation among the continuum of Pareto efficient allocations) can always be achieved *under market conditions*, when the government uses lump-sum taxes and transfers to redistribute wealth. This is a result of the "Second Fundamental Theorem of Welfare Economics", which states that to every Pareto efficient allocation, there corresponds a competitive market equilibrium, obtained under a particular distribution of initial endowments and, thus, wealth ("wealth" is the value of a consumer's initial endowment, evaluated at market prices).

What do we learn from this? If the assumptions of the neoclassical theory are (approximately) fulfilled, then the government must do only two things to maximize social welfare:

1. make sure that perfect competition prevails in all markets (hence, establish a functioning market economy): this assures Pareto efficiency.
2. redistribute wealth for equity reasons to achieve not only economic efficiency, but also social optimality.

To put this in other words: theory suggests that, in policy analysis, problems of efficiency and equity can be dealt with *independently*. Furthermore, whenever economists think that there is a "problem" (e.g. global warming), it should be possible to identify one or several underlying market failures, because the assumptions of the neoclassical theory are, then, violated. Otherwise, there would be no "problem", and perfect competition would automatically lead to an efficient allocation. Hence, in this course on environmental economics, we will often try to identify or characterize the reasons why an unregulated market economy does *not* lead to efficiency (environmental externalities are one possible source of market failure, but there are many others). Furthermore, we will try to find out how a regulator can correct for these market failures, in order to achieve an efficient or optimal outcome, e.g. by using taxes. By contrast, we will not focus on equity issues. Relying on the Second Fundamental Theorem of Welfare Economics, we assert that these issues can be dealt with separately (within a functioning welfare state). Our focus is, thus, on efficiency.

Remember: if there is no market failure, then there is no reason why the government should do *anything* (except redistributing wealth for equity reasons). Therefore, as economists, we need to identify sources of market failure in order to justify a policy intervention.

2.1 Welfare economics and efficiency in the static framework

Let us review the conditions of efficiency in a static framework (many of these things may be familiar to you from previous courses, but we start from scratch here so we do not require you to know them already).

For illustrative purposes, let us assume in the following that there are only two individuals in our model economy (A and B), two goods or services (consumption quantities X and Y), and two inputs or resources K and L (think e.g. of capital and labor) that exist in fixed quantities (this is the endowment of the economy).[14] We assume away any externalities (hence, consumption or production of a good has no positive or negative side-effects on other consumers or firms). In addition to that, we assume that all goods are private (not public), hence, if somebody owns a commodity (has property rights over it), nobody else can consume it or use it for production.

Suppose, that preferences over bundles of goods can be represented by utility functions:

$$U^A = U^A(X^A, Y^A) \quad , \quad U^B = U^B(X^B, Y^B).$$

E.g., $U^A(X^A, Y^A)$ is consumer A's utility when consuming the quantities X^A resp. Y^A of good X resp. of good Y.

If the *output* quantity X depends only on the input quantities used in the production of this output ($K^X$and L^X: capital and labor employed in the sector producing good X), and similarly for output Y, the technological possibilities can be expressed by the following production functions (assuming efficient production):

$$X = X(K^X, L^X) \quad , \quad Y = Y(K^Y, L^Y).$$

When there are several firms, we may assume that all firms have the same technology. However, here, we look at efficiency at a more abstract level, and do not discuss any specific institutional arrangements (such as the existence of firms or markets).

[14] Everything that we explain in the following extends readily to an economy with a large number of consumers, goods, sectors etc..

3 Market failure, public goods, externalities

3.1 Market failure, public goods

We already mentioned conditions that must hold in an "ideal" market economy, under which the market outcome coincides with the allocation that a social planner would choose, hence, the "social optimum". If any of these conditions is not fulfilled, the market outcome will be inefficient, and we say there is a "market failure". Let us summarize these ideal conditions:

1. Markets exist for all goods and services produced and consumed
2. All markets are perfectly competitive (price-taking firms, no monopoly power)
3. All actors have perfect information
4. Private property rights are fully assigned in all resources and commodities
5. No externalities exist
6. All goods and services are private goods (there are no "public goods")
7. All utility and production functions are "well-behaved"
8. All agents are maximizers (utility/profit maximizers).

Let us briefly discuss a few examples where one or several of the above conditions are violated. For example, private property rights often do not exist for renewable resources. An example for this is ocean fishery. If anyone can go out and fish, the exploitation of this resource is uncontrolled, and there will generally be over-exploitation. Another example are "stock-pollution problems", where the earth or the atmosphere is effectively used as a waste sink, e.g. for carbon dioxide emissions. Generally, no private property rights are assigned, so the atmosphere is an open-access resource. Note, that climate change or (local) air pollution causes a negative externality.

An important distinction that is often made in the literature is between private and public goods. Private goods are characterized by rivalry and excludability, where rivalry refers to whether one agent's consumption is at the expense of another agent's consumption (think, e.g., of ice cream) and excludability refers to whether agents can be prevented from consuming.

A "pure" public good is e.g. national defense. No citizen can be excluded from enjoying the benefits of it, and the consumption is clearly non-rival. Some public goods are non-rival, but excludable. They are usually referred to as "congestible resources". An example are wilderness areas. Enjoying wilderness by one individual is generally not at the expense of another agent's joy (unless it is used to such an extent that congestion occurs). However, using fences, individuals can be excluded from consumption. Other public goods are rival in consumption, but non-excludable. They are usually referred to as "open-access resources". An example is the ocean-fishery.

Consumption is rival, because each fish can only be caught once, so the more boats are fishing, the harder it gets to fish. However, it is difficult to exclude anyone from fishing.

Let us now analyze the efficient allocation of a public good formally. To this end, we go back to our earlier model economy with two consumers (A and B), and two goods (X and Y). The top-level product-mix condition for allocative efficiency was given by:

$$MRUS^A = MRUS^B = MRT.$$

Intuitively, MRT (marginal rate of transformation) describes how (given an efficient use of the inputs to production) good Y can be transformed into good X, and vice versa. MRT describes how costly the production of good X is in terms of forgone output of good Y.

$MRUS$ (marginal rate of utility substitution) describes at what rate a consumer would exchange good Y for good X, given that the consumer's utility remains unchanged. $MRUS$ describes how valuable the consumption of good X is in terms of forgone consumption of good Y. Efficiency requires that these relative benefits / costs must be equal at the margin. Otherwise, production or consumption of one of the goods may be raised, and the consumer who becomes better off can compensate the consumer who gets worse off such that both consumers benefit from the voluntary exchange (hence, a Pareto improvement is possible).

Now suppose, X is a *public good*, and Y is a private good. This means that consumers do no longer care about their individual consumption of good X, but only about the aggregate consumption: $X = X^A + X^B$. Hence, the utility functions are given by:

$$U^A = U^A(X, Y^A), \quad U^B = U^B(X, Y^B)\ , \ \text{ where } X = X^A + X^B.$$

It can be shown that in this case, the above top-level efficiency condition becomes:

$$MRUS^A + MRUS^B = MRT.$$

Intuitively, this means that for efficiency, not the individual marginal benefit of consumption of good X matters, but the aggregated marginal benefit of all consumers. In general, this implies that more of good X will be provided in the optimum than under market conditions (the market provides too little of the public good – the reverse holds true if X is a public bad, such as pollution).

The derivation and interpretation of the above efficiency condition is easier in the context of a partial equilibrium model. Let us go through the details. In a partial equilibrium model, the utility functions are quasi-linear, and can be written as (the superscript P stands for Partial equilibrium model):

$$U^A = U^{P,A}(X) + Y^A \quad , \quad U^B = U^{P,B}(X) + Y^B.$$

Let $Y = f(X)$ be the production possibility frontier (combinations of X and Y that the economy can generate, given an efficient use of the available resources). To determine the efficiency condition, set up the Lagrangian. The target function is $U^{P,A}$, A's utility, while B's utility is held

Furthermore, abatement costs are not constant over time, but depend on the rate of technological change. Hence, if a high carbon price is implemented early on, future abatement costs are likely to be lower due to induced technological change. Therefore, it is not clear whether the "Weitzman-argument" is still valid in a dynamic environment, and in fact, there is strong evidence that it is not – hence, carbon taxes may be preferable even when there is a risk of catastrophic climate change.

For example, a carbon tax may be used early on, which gives price stability for investors in low-carbon technologies. If the resulting emissions reductions are insufficient, the regulator can later still implement a cap&trade scheme if necessary. Newell and Pizer (2003) find that the expected welfare gains under such a dual approach are considerably higher than under a pure cap&trade scheme that is implemented right from the start.

5.4 Exercises

Problem 1 (Input mix under a fossil fuel tax)

Consider a firm that uses two inputs (input-quantities z_1,z_2 , input 2 is a fossil fuel). The output quantity is q, and the production function is:

$$q = f(z_1, z_2) = 2\sqrt{z_1} + \sqrt{z_2}.$$

The output price is p. The input price w_1 is equal to 1. w_2 is the sum of the market price for the input: $\widetilde{w}_2 = 3$, and the carbon tax t, hence: $w_2 = 3 + t$.

a) Write down the firm's profit maximization problem.

b) Solve the profit maximization problem to compute the optimal input mix.

c) Compute the firm's supply function $q(p)$.

Now suppose, 10 identical firms with the same supply function $q(p)$ are in this competitive market. Market demand is given by $D(p) = 22 - p$.

d) What is the aggregate supply function? Sketch the market demand and supply function for a tax rate of $t = 2$ in a p-q-diagram, and compute the market equilibrium. In your figure, illustrate the effects of an increase in the tax rate t. State welfare effects of the tax.

Problem 2 (Prices vs. quantities)

Consider an economy characterized by the following abatement cost and benefit functions:

$$C(A,\theta) = \frac{A^2}{2} + \theta A \text{ , and } \quad B(A) = 2A - \frac{A^2}{4}$$

A is the abatement of emissions, and θ is a shock that affects the abatement costs. Suppose, θ can take on only two values: $\theta \in \{-1,1\}$, with $Pr[\theta = 1] = 1/2$.

a) Derive the marginal abatement cost and benefit functions (MC, MB). What is the expected marginal cost function (MC^e)?

b) Compute the (*ex-ante*) optimal quantity instrument $\hat{A}$ and the optimal price instrument $\tilde{p}$.

c) Consider a positive cost shock: $\theta = +1$. Carefully plot the functions MC, MB, and MC^e in one diagram (drawn roughly to scale). Indicate in the diagram the location of the optimal abatement quantity *given* the shock θ, and the location of the (*ex-ante*) optimal quantity instrument $\hat{A}$. Is the optimal abatement *given* the shock θ higher or lower than $\hat{A}$? Why?

d) Compare the net surplus in a situation where the *ex-post* optimal quantity is chosen (hence, optimal for the given θ) with the net surplus under the *ex-ante* optimal quantity instrument $\hat{A}$. In your diagram, indicate the welfare loss (= difference in net benefit between these two cases) that arises due to the regulator's *ex-ante* uncertainty about θ.

e) Add to your diagram the location of the (*ex-ante*) optimal price instrument $\tilde{p}$. Indicate in the diagram the location of the *actual* abatement quantity under the price policy $\tilde{p}$. Is the actual abatement under the price policy $\tilde{p}$ higher or lower than the optimal abatement quantity *given* the shock θ? Why?

f) Compare the net surplus in a situation where the *ex-post* optimal quantity is chosen with the net surplus under the *ex-ante* optimal price instrument $\tilde{p}$. In your diagram, indicate the welfare loss (= difference in net benefit between these two cases) that arises due to the regulator's *ex-ante* uncertainty about θ.

g) Is the welfare loss (graphically) higher or lower under the price policy, compared to the case with the quantity instrument? Check your result qualitatively using the formula $\Delta = \frac{\sigma^2}{2\beta^2}(\beta + \delta)$.

Solutions

Problem 1

a) $\underset{z_1,z_2}{max}\pi(z_1,z_2) = p \cdot (2\sqrt{z_1} + \sqrt{z_2}) - z_1 - (3+t)z_2$

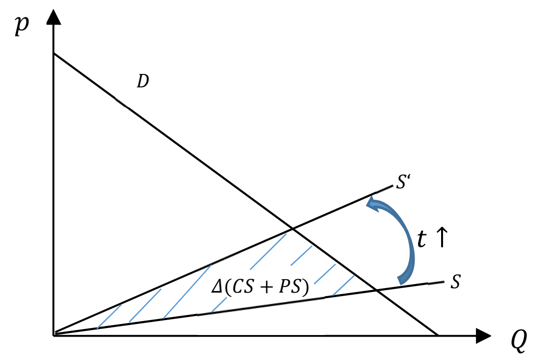
b) FOCs: $0 = \frac{2p}{2\sqrt{z_1}} - 1 \rightarrow z_1^* = p^2$

$0 = \frac{p}{2\sqrt{z_2}} - (3+t) \rightarrow z_2^* = \frac{p^2}{4(3+t)^2}$ (factor demand functions)

c) Supply function: $q(p,t) = f(z_1^*, z_2^*) = 2p + \frac{p}{2(3+t)}$

d) Aggregate supply function: $S(p,t) = 20p + \frac{5p}{3+t}$

Market equilibrium for $t = 2$: $D(p) = S(p,t) \rightarrow 22 - p = 21p \rightarrow p^* = 1$, $Q^* = D(p^*) = 21$



Welfare effects: reduction of consumer surplus (negative), reduction of producer surplus (negative), increase in tax revenues (positive), benefits of mitigated climate change (positive)

Problem 2

a) The marginal abatement cost and benefit functions are:

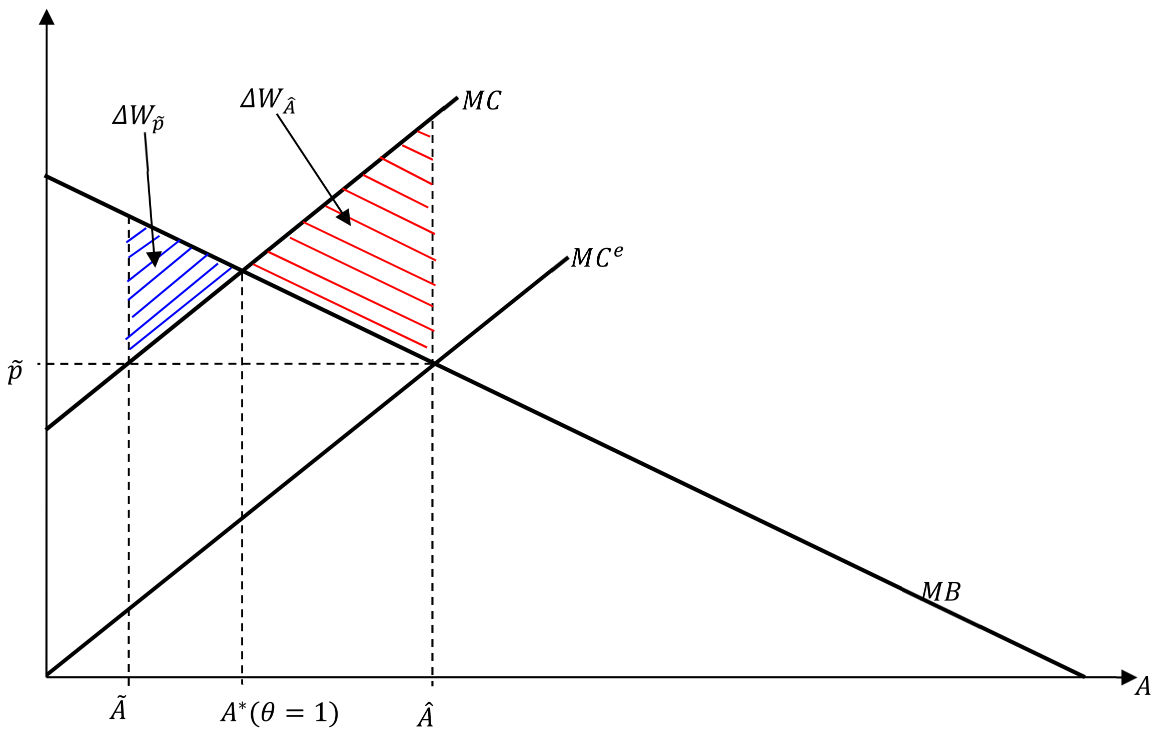
$$MC = \frac{\partial C(A,\theta)}{\partial A} = A + \theta \ , \quad \text{and} \quad MB = B'(A) = 2 - \frac{A}{2}$$

The expected marginal cost function is $MC^e = E[MC] = A$.

b) The optimal quantity instrument solves: $MB = MC^e$, hence: $A = 2 - \frac{A}{2} \rightarrow \hat{A} = 4/3$.

The optimal price instrument is given by: $\tilde{p} = MB(\hat{A}) = 2 - 2/3 = 4/3$.

c) – g) are solved using the following diagram:



c) The optimal abatement given θ (in the diagram indicated by $A^*(\theta = 1)$) is lower than the *ex-ante* optimal quantity instrument $\hat{A}$, because the marginal costs are higher than expected by the regulator; therefore, it is optimal to undertake less abatement when θ is known.

d) The welfare loss under the quantity instrument is the shaded area $\Delta W_{\hat{A}}$. This welfare loss reflects the additional costs incurred net of the additional benefits when the abatement is $\hat{A}$ rather than $A^*(\theta = 1)$.

e) The actual abatement under the price policy $\tilde{p}$ (in the diagram indicated by $\tilde{A}$) is lower than the optimal abatement quantity given θ ($A^*(\theta = 1)$). This is because the price is fixed at a level that is too low, compared to the optimal price level under θ. The reason is that $\tilde{p}$ reflects the expected marginal cost, but the actual marginal cost is higher. As a result, firms undertake *too little abatement* under the price instrument.

f) The welfare loss under the price instrument is the shaded area $\Delta W_{\tilde{p}}$. This welfare loss reflects the additional benefits of abatement (net of costs) that could be achieved by doing more abatement.

g) The welfare loss is higher under the quantity instrument. This is confirmed by the formula, because $|\delta| < \beta$ holds (using $\beta = 1$ and $\delta = -1/2$), hence, the marginal benefit curve is less steep than the marginal cost curve.

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Part 2:
Resource Economics

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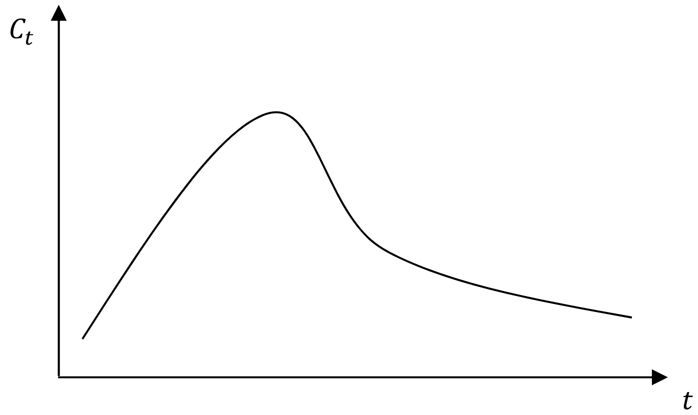
The following figure shows a typical consumption path that emerges from this type of model:



As the figure shows, consumption first increases as capital is accumulated, but eventually, as the stock of the non-renewable resource is exhausted, it starts to decline again and asymptotically goes to zero. For $\rho > 0$, this holds even if the production technology is such that a constant level of consumption could be maintained forever (this is not true for all production functions $Q(K_t, R_t)$).

The lesson from this prediction is, that discounting can lead to a situation where the current generation is better off, while many future generations are worse off, and consumption may even approach zero, even if a positive consumption level could be maintained forever. This is related to the issue of sustainability, to which we will return later during this course.

For the moment, keep in mind that utilitarianism can lead to results that many would consider as being unfair towards future generations.

As a technical aside to which we will return to later, note that intertemporal efficiency is trivially fulfilled in the above growth model with only capital, but an intertemporal efficiency condition arises in the model with the resource. In addition to the trade-off between current consumption and capital investments, there is now the possibility of reducing the current rate of resource depletion so as to leave more of it for future use. The additional intertemporal efficiency condition that arises requires the equalization of the rates of return to capital accumulation and resource conservation (see below for further details).

2.2 Introduction to dynamic optimization

Optimal control theory, using the maximum principle, is a technique for solving constrained dynamic optimization problems, such as the ones outlined above. Proofs of the optimality conditions are not given here, but you should learn how to use the technique, using these notes.

We introduce the technique heuristically, by means of an example (below). If you are interested to go deeper into this mathematical concept, please refer to textbooks[1] and other sources. We only use the most basic concepts, sufficient for our purposes in this course. Our goal is to derive optimality conditions for dynamic optimization problems, that mirror the optimality conditions in static constrained optimization problems, using the Lagrangian (see the first part of this course where we repeated this technique).

Consider the following dynamic optimization problem. It is a simple optimal growth model:

$$\max_{\{C_t\}} \int_0^\infty U(C_t)e^{-\rho t}dt \text{ s.t. } \dot{K}_t = Q(K_t) - C_t.$$

In words: a social planner seeks to optimize the consumption and savings decisions at each point in time, over an infinite time horizon. Utility at time t is $U(C_t)$, where C_t is consumption at t. The discount factor to evaluate future utility is $e^{-\rho t}$, where ρ is the utility discount rate (or pure rate of time preference). The second part, $\dot{K}_t = Q(K_t) - C_t$, is the law of motion for the capital stock, K_t. The output of the economy at time t, K_t, can either be consumed, or invested. The investment is $I_t = \dot{K}_t$ (assuming away capital depreciation for simplicity), so this condition is equivalent to the identity: $Q(K_t) = C_t + I_t$, which says that the output of the economy can be used for consumption and for investment (and nothing else).

To solve an optimal control problem such as the one above, the first thing to note is that there are two types of variables involved: state variables and control variables. *State variables* are variables that generally change *gradually* over time. They typically reflect *stocks* of some commodities or resources. For example, the stock of pollution in the atmosphere is a typical state variable. If pollution continues, this stock gradually grows over time, and if there is no more pollution, then due to some natural decay of the pollutant, this stock may also decline again over time. In the above problem, there is only one state variable: the capital stock K_t.

The other type of variables that are part of a dynamic optimization problem are the *control variables*. These are the variables that the planner can control more or less directly, and that – in principle – may be changed instantaneously at any point in time (and – unlike the state variables – not only gradually). In a pollution problem, a control variable might for example be the rate at which fossil fuels are burned at time t. This rate, then, affects the evolution of the state variable (the pollution stock) over time, which changes gradually. Control variables are typically flow variables. These may literally be some flows of pollutants or resources, or other variables that can be changed instantaneously by the planner (if she wishes to do so).

In the above problem, there is only one control variable: the consumption flow at time t, C_t. The smaller is C_t, for a given output of the economy at time t, $Q(K_t)$, the more is invested in the capital stock. This means that current consumption is reduced, but in exchange, more can be

[1] E.g., Kamien and Schwartz (2012).

produced (and consumed) in the future, as the capital stock then grows more rapidly. This is the trade-off in an optimal growth problem such as the one above.

The first step in the dynamic optimization procedure is to define the current-value Hamiltonian that corresponds to this problem. (This mirrors the Lagrangian function in a static constrained optimization problem). For the above problem, it is defined as follows:

$$H_{C_t} = U(C_t) + \mu_t(Q(K_t) - C_t) = U(C_t) + \mu_t \dot{K}_t.$$

Here, the Hamiltonian is current utility plus the increase in the capital stock, valued using the shadow price of capital. The shadow price of capital is μ_t. This variable is defined as part of the dynamic optimization procedure, but it has a useful economic interpretation (see below). It is very similar to the Lagrangian parameter λ that we define in a static constrained optimization problem. In a dynamic optimization problem, this shadow price is also called a "co-state variable". This indicates that one co-state variable is defined for each constraint in the optimization problem. In the problem defined above, the constraint is the law of motion for the capital stock, that captures how the capital stock changes over time ($\dot{K}_t = Q(K_t) - C_t$), depending on the control variable C_t. Because this constraint relates to the capital stock, the corresponding co-state variable (here: μ_t) also says something about the value of capital (from the planner's perspective). Below, we will explain why it is, therefore, also referred to as the "shadow price" (or shadow value) of capital.

The maximum principle condition in the dynamic optimization procedure is a theoretical concept. We skip the general mathematical proofs underlying this procedure, and only show heuristically how the procedure is applied (as we already mentioned above). Here, this principle leads to the following so-called "static efficiency condition":

$$\frac{\partial H_{C_t}}{\partial C_t} = \frac{\partial U(C_t)}{\partial C_t} - \mu_t = 0,$$

Hence: $\mu_t = \frac{\partial U(C_t)}{\partial C_t}$. In words: the shadow price of capital, μ_t, is at every point in time t (along the optimal path) equal to the marginal utility of consumption.

In general (as a "cooking recipe" for the dynamic optimization procedure), each of the static optimality condition(s) is always derived as follows: take the current-value Hamiltonian, and take the derivative of it w.r.t. each of the control variables, equalizing it to zero. For the above problem, this yields: $\frac{\partial H_{C_t}}{\partial C_t} = 0$, and, after evaluating this derivative, we arrive at the condition $\mu_t = \frac{\partial U(C_t)}{\partial C_t}$.

This condition is quite intuitive, because a marginal addition to the capital stock comes at the cost of a marginal reduction in consumption (at the same instant of time). Hence, we can also interpret this static efficiency condition as a balance of marginal benefits and marginal costs. Here, the benefits of increasing consumption at time t are given by the marginal utility of consumption (at time t, because we use the current value Hamiltonian). This marginal benefit is equalized with the