On the Reducibility of Persistent Petri Nets

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Abstract

This paper aims to find a transformation from persistent Petri nets, which are a general class of conflict-free Petri nets, into a more restricted class of nets called behaviourally conflict-free nets. In a persistent net, whenever two distinct transitions are simultaneously enabled, one cannot become disabled through the occurrence of the other. In a behaviourally conflict-free net, two distinct transitions which are simultaneously enabled do not share a common pre-place. Relying on a series of earlier results which characterise the cyclic structure of the reachability graphs of persistent nets, we present a partial solution for transforming persistent into behaviourally conflict-free nets.

1 Introduction

There exists a hierarchy of Petri net classes [3], all of which can intuitively be called 'conflict-free', and of which marked graphs [5, 6] are the smallest and persistent nets [8] the largest class. This paper is concerned with an intermediate class called *behaviourally conflict-free* Petri nets. We address the question whether a persistent net can be transformed into a behaviourally conflict-free net with isomorphic reachability graph. We prove that under some conditions, such a transformation can be found.

2 Definitions

A Petri net (S, T, F, M_0) consists of two finite and disjoint sets S (places) and T (transitions), a function $F\colon ((S \times T) \cup (T \times S)) \to \mathbb{N}$ (flow) and a marking M_0 (the initial marking). A marking is a mapping $M\colon S \to \mathbb{N}$. A Petri net is plain if the range of F is $\{0, 1\}$, i.e., F is a relation. The pre-set ${}^\bullet x$ of a net element $x \in S \cup T$ is the set $\{y \in S \cup T \mid (y, x) \in F\}$. Similarly, the post-set of x is $x^\bullet = \{y \in S \cup T \mid (x, y) \in F\}$.

The incidence matrix C is an $S \times T$-matrix of integers where the entry corresponding to a place s and a transition t is, by definition, equal to the number $F(t,s) - F(s,t)$. A T-invariant J is a vector of integers with index set T satisfying $C \cdot J = \underline{0}$ where $\cdot$ is the inner (scalar) product, and $\underline{0}$ is the vector of zeros with index set S. J is called semipositive if $J(t) \geq 0$, for all $t \in T$. The support of a semipositive T-invariant J, written $supp(J)$, is the set of transitions t for which $J(t) > 0$. Two semipositive T-invariants J and J' are called transition-disjoint if $\forall t \in T$: $J(t) = 0 \vee J'(t) = 0$, or, equivalently, if $supp(J) \cap supp(J') = \emptyset$. For a sequence $\sigma \in T^*$ of transitions, the Parikh vector $\Psi(\sigma)$ is a vector of natural numbers with index set T, where $\Psi(\sigma)(t)$ equals $\#(t,\sigma)$, the number of occurrences of t in σ.

A transition t is enabled (or activated, or firable) in a marking M (denoted by $M[t\rangle$) if, for all places s, $M(s) \geq F(s,t)$. If t is enabled in M, then t can occur (or fire) in M, leading to the marking M' defined by $M'(s) = M(s) + F(t,s) - F(s,t)$ (notation: $M[t\rangle M'$). We apply definitions of enabledness and of the reachability relation to transition (or firing) sequences $\sigma \in T^*$, defined inductively: $M[\varepsilon\rangle$ and $M[\varepsilon\rangle M$ are always true; and $M[\sigma t\rangle$ (or $M[\sigma t\rangle M'$) iff there is some M'' with $M[\sigma\rangle M''$ and $M''[t\rangle$ (or $M''[t\rangle M'$, respectively).

A marking M is reachable (from M_0) if there exists a transition sequence σ such that $M_0[\sigma\rangle M$. The reachability graph of N, with initial marking M_0, is the graph whose vertices are the markings reachable from M_0 and where an edge (M,t,M') labelled with t leads from M to M', iff $M[t\rangle M'$. Figure 1 shows an example where on the right-hand side, M_0 denotes the marking shown in the Petri net on the left-hand side. The marking equation states that if $M[\sigma\rangle M'$, then $M' = M + C \cdot \Psi(\sigma)$. Thus, if $M[\sigma\rangle M$ then $\Psi(\sigma)$ is a T-invariant.

A Petri net with initial marking is k-bounded if in any reachable marking M, $M(s) \leq k$ holds for every place s, and bounded if there is some k such that it is k-bounded. A finite Petri net (and we consider only such nets in the sequel) is bounded if and only if the set of its reachable markings is finite. A net with initial marking is called reversible if its reachability graph is strongly connected.

A net $N = (S,T,F,M_0)$ is a marked graph if $\sum_{t \in T} F(s,t) \leq 1$ as well as $\sum_{t \in T} F(t,s) \leq 1$, for all places s; output-nonbranching (on) if $\sum_{t \in T} F(s,t) \leq 1$ for all places s; behaviourally conflict-free (bcf) if, whenever $M[t_1\rangle$ and $M[t_2\rangle$ for a reachable marking M and transitions $t_1 \neq t_2$, then ${}^\bullet t_1 \cap {}^\bullet t_2 = \emptyset$; and persistent, if whenever $M[t_1\rangle$ and $M[t_2\rangle$ for a reachable marking M and transitions $t_1 \neq t_2$, then $M[t_1 t_2\rangle$. Directly from this definition, we have

$$\text{marked graph} \;\Rightarrow\; \text{on} \;\Rightarrow\; \text{bcf} \;\Rightarrow\; \text{persistent}.$$

The aim of this paper is to show that under certain conditions, the last implication can 'essentially' be reversed. Consider two different transitions in the post-set of a place and a reachable marking. If the net is persistent, then

(a) either at most one of the transitions is enabled at the marking or
(b) both are enabled but the occurrence of any of the transitions does not disable the other transition (which is in particular the case if the transitions are concurrently enabled).

If the net is behaviourally conflict-free, then (b) is not possible, i.e., at most one of the transitions is enabled at the marking. So our aim is to investigate conditions, under which forward branching places with option (b) can be replaced in such a way that the resulting net has only forward branching places with option (a).

Throughout the paper, we assume all nets to be plain (no arc weights > 1), T-restricted (transitions have at least one input place and at least one output place), simply live (every transition can be fired at least once), and free of isolated places.

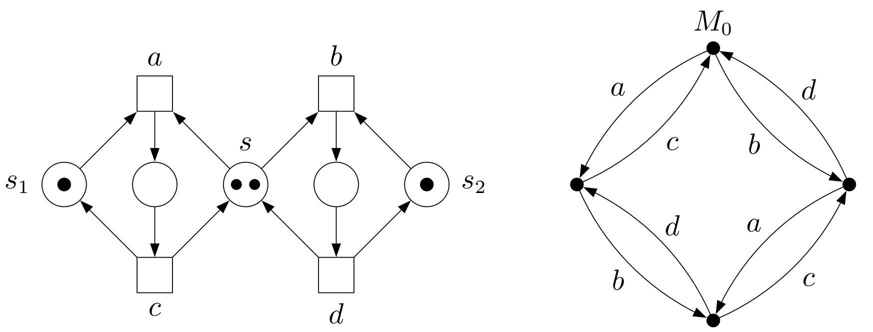

Figure 1: A persistent Petri net and its reachability graph

Figure 1 shows a Petri net which is persistent but not behaviourally conflict-free. Initially, a and b are enabled and share a common pre-place s. Observe that in the special case shown in this figure, place s can actually just be omitted without altering the net's behaviour. *Unaltered behaviour* here (and in the following) means reachability graph isomorphism. Alternatively, s could be split into two separate places, with one (or even two) tokens each, one of them connected only to the left-hand-side cycle, the other connected to the right-hand-side cycle. In both cases, the net is reduced to a (disconnected) marked graph. We investigate general circumstances under which such transformations are possible.

Two sequences $M[\sigma\rangle$ and $M[\sigma'\rangle$, firable from M and with $\sigma, \sigma' \in T^*$, are said to arise from each other by a transposition if they are the same, except for the order of an adjacent pair of transitions, thus:

$$\sigma \quad = \quad t_1 \dots t_k t t' \dots t_n \quad \text{and} \quad \sigma' \quad = \quad t_1 \dots t_k t' t \dots t_n.$$

Two sequences $M[\sigma\rangle$ and $M[\sigma'\rangle$ are said to be permutations of each other (from M, written $\sigma \equiv_M \sigma'$) if they are both firable at M and arise out of each other through a (possibly empty) sequence of transpositions.

Theorem 1. Keller [7]

Let N be a persistent net and let $\tau_1, \dots, \tau_m$ be m firing sequences starting from some reachable marking M. Then there is also a firing sequence $M[\tau\rangle$ such that $\forall t \in T \colon \Psi(\tau)(t) = \max_{1 \le j \le m} \Psi(\tau_j)(t)$.

A transition sequence $\tau \in T^*$ is called cyclic if its Parikh vector is a T-invariant (which is the case if and only if for all markings M, $M[\tau\rangle$ implies $M[\tau\rangle M$).

A cyclic transition sequence τ is called decomposable if $\tau = \tau_1\tau_2$ such that τ_1 and τ_2 are cyclic and $\tau_1 \neq \varepsilon \neq \tau_2$. A firing sequence $M[\tau\rangle M'$ is called a cycle if τ is cyclic, i.e. if $M = M'$. A cycle $M[\tau\rangle M$ is called simple if there is no permutation $\tau' \equiv_M \tau$ such that τ' is decomposable. In other words, a non-simple sequence can be permuted such that the permuted sequence has a smaller cyclic subsequence leading from some marking back to the same marking. In Figure 1, for example, we have that:

$abcd$	is not decomposable
$acbd$	is decomposable, namely by $\tau_1 = ac$ and $\tau_2 = bd$
$M_0[ac\rangle M_0$	is simple
$M_0[abcd\rangle M_0$	is not simple, because of the permutation $M_0[acbd\rangle M_0$.

Figure 2 shows that a simple cycle can contain some transition more than once.

Theorem 2. A decomposition theorem [1, 2]

Let N, with some initial marking, be bounded, reversible, and persistent. There is a finite set $\mathcal{B}$ of semipositive T-invariants such that any two of them are transition-disjoint and every cycle $M[\alpha\rangle M$ in the reachability graph decomposes up to permutations to some sequence of cycles $M[\alpha_1\rangle M[\alpha_2\rangle M \dots [\alpha_n\rangle M$ with all Parikh vectors $\Psi(\alpha_i) \in \mathcal{B}$.

$\mathcal{B}$ can be constructed by picking simple cycles in the reachability graph and computing their T-invariants. For example, for Figure 1,

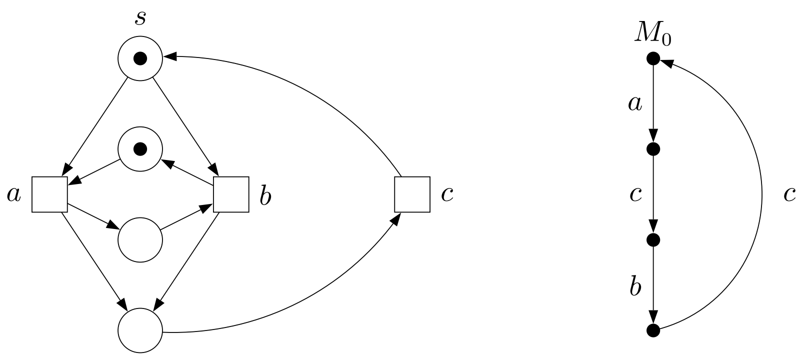

Figure 2: Another persistent net and its reachability graph

$\mathcal{B} = \{J_1, J_2\}$ with

$$J_1(a){=}J_1(c){=}1,\ J_1(b){=}J_1(d){=}0;\ J_2(a){=}J_2(c){=}0,\ J_2(b){=}J_2(d){=}1.$$

For Figure 2, $\mathcal{B} = \{J\}$ with $J(a){=}1$, $J(b){=}1$, and $J(c){=}2$.

Theorem 3. A CONVERSE OF THEOREM 2 [2]

Let N, with some initial marking, be bounded, reversible, and persistent. Let M be a reachable marking. Let $J_1, \ldots, J_m$ be (not necessarily mutually distinct) T-invariants from $\mathcal{B}$, as in Theorem 2. Then there is a cycle $M[\alpha\rangle M$ such that $\alpha = \alpha_1 \ldots \alpha_m$ and $\Psi(\alpha_j) = J_j$, for all $1 \leq j \leq m$.

Thus, the reachability graph is covered by simple cycles, any two of which are either transition-disjoint or have the same Parikh vector. Moreover, any such simple cycle can be fired from anywhere in the reachability graph, though the order of firing its transitions may vary from marking to marking. It is shown in [2] that reversibility is absolutely needed for these strong results, but that part of the decomposition properties can be recovered if the premise of reversibility is dropped.

4 Analysing branching places

From now on, let N be a bounded, reversible, persistent net with initial marking M_0.

Let us consider a place s and the set $s^\bullet$ of its output transitions. By the absence of isolated places and dead transitions and by reversibility, $s^\bullet \neq \emptyset \neq {}^\bullet s$ and every $a \in s^\bullet$ is in one of the simple cycles mentioned in the above theorem. Let $\mathcal{J}_s$ denote the set of T-invariants in $\mathcal{B}$ such that every one of them contains some transition in $s^\bullet$. We distinguish two cases.

Case 1:

$|\mathcal{J}_s| = 1$, that is, there is a simple cycle of the reachability graph containing *all* transitions in $s^\bullet$. This is certainly the case if $|s^\bullet| = 1$, but it may also be the case if $|s^\bullet| > 1$, as shown in Figure 2. As a consequence of the decomposability theorem, the simple cycle γ containing all transitions in $s^\bullet$ must also contain all transitions in ${}^\bullet s$ since if some of them were missing, there would be another simple cycle through them, which would have to contain at least one transition in $s^\bullet$ in order to restore the marking on s and would thus not be transition-disjoint with γ. This case will be investigated in section 5 below.

Case 2:

$|\mathcal{J}_s| > 1$, that is, all simple cycles contain proper subsets of $s^\bullet$, and therefore (with the same reasoning as above), also proper subsets of ${}^\bullet s$. An example is shown in Figure 1 where $|\mathcal{J}_s| = 2$.

Suppose that $\mathcal{J}_s = \{J_1, \ldots, J_m\}$ with $m > 1$. Our decomposition theorems imply that the set $\{supp(J_\ell) \cap s^\bullet \mid 1 \leq \ell \leq m\}$ partitions the set $s^\bullet$ and the set $\{supp(J_\ell) \cap {}^\bullet s \mid 1 \leq \ell \leq m\}$ partitions the set ${}^\bullet s$. Moreover, none of the sets in these sets is empty (which implies, in particular, that both $s^\bullet$ and ${}^\bullet s$ contain at least m transitions).

Now we will make a connection between finite *synchronic distances* and places. Informally, the (asymmetric) synchronic distance between two sets of transitions A and B indicates how far transitions from A can 'run ahead' of transitions from B. Formally:

$$asd_M(A,B) = \max\{\big(\sum_{t_1 \in A} \#(t_1,\tau)\big) - \big(\sum_{t_2 \in B} \#(t_2,\tau)\big) \mid \tau \in T^*,\ M[\tau\rangle\}.$$

If some sequence τ with $M[\tau\rangle$ actually satisfies

$$asd_M(A,B) = \big(\sum_{t_1 \in A} \#(t_1,\tau)\big) - \big(\sum_{t_2 \in B} \#(t_2,\tau)\big),$$

then we call it a *witness* for the 'gap' $asd_M(A,B)$.

The maximum can become infinite. For instance, $asd_{M_0}(\{a\},\{d\}) = \infty$ in Figure 1 and $asd_{M_0}(\{c\},\{a\}) = \infty$ in Figure 2. If the sets A and B are 'controlled' by place s and transition invariant J_ℓ, however, then their synchronic distance is always finite.

Lemma 4. Controlled transition sets by induce finite *asd*

Let N be a bounded, reversible, persistent net with initial marking M_0, let s be a place of N, let $J \in \mathcal{J}_s$ and let $A = supp(J) \cap (s^\bullet \setminus {}^\bullet s)$ and $B = supp(J) \cap ({}^\bullet s \setminus s^\bullet)$. Then both $asd_{M_0}(A,B)$ and $asd_{M_0}(B,A)$ are well-defined finite numbers.

Proof: By Theorem 3, there exists an infinite firing sequence

$$M_0[\tau\rangle M_0[\tau\rangle M_0[\tau\rangle M_0 \ \ldots$$

such that $\Psi(\tau) = J$. In this sequence, the only transitions putting tokens on s are those of B and the only transitions removing tokens from s are those of A. In case $asd_{M_0}(A, B) = \infty$, we get a contradiction to the fact that s contains finitely many tokens initially, and in case $asd_{M_0}(B, A) = \infty$ we get a contradiction to the boundedness of s. □ 4

Using this lemma, and keeping $\mathcal{J}_s = \{J_1, \ldots, J_m\}$ in mind, we can define the numbers

$$L_\ell \quad = \quad asd_{M_0}(supp(J_\ell) \cap (s^\bullet \setminus {}^\bullet s), supp(J_\ell) \cap ({}^\bullet s \setminus s^\bullet))$$

for every $1 \le \ell \le m$. Let the net $N[s] = (T', S', F', M_0')$ be defined from $N = (S, T, F, M_0)$ as follows.

- The transitions of $N[s]$ are $T' = T$, the same as the transitions of N.
- The places of $N[s]$ are $S' = S \uplus \{s_1, \ldots, s_m\}$, i.e. the places of N plus m new places s_ℓ, one for each $\ell \in \{1, \ldots, m\}$.
- The initial marking M_0' is defined as follows: $M_0'(q) = M_0(q)$ for every $q \in S$, and $M_0'(s_\ell) = L_\ell$ for every $\ell \in \{1, \ldots, m\}$.
- The flow relation is extended as follows:

$$\begin{aligned} F' \quad = \quad F \quad & \cup \{(s_\ell, t) \mid t \in supp(J_\ell) \cap s^\bullet, 1 \le \ell \le m\} \\ & \cup \{(t, s_\ell) \mid t \in supp(J_\ell) \cap {}^\bullet s, 1 \le \ell \le m\}. \end{aligned}$$

Theorem 5. Place covering

Let N be a bounded, reversible, persistent net with initial marking M_0, let s be a place with $|\mathcal{J}_s| > 1$ and let $N[s]$ be constructed as above. Then the reachability graphs of N and $N[s]$ are isomorphic. Moreover, with the numbers $L_1, \ldots, L_m$ defined above, we have $L_1 + \ldots + L_m \le M_0(s)$.

Proof: For reachability graph isomorphism, we first note that by construction, every firing sequence of $N[s]$ is also a firing sequence of N (this is always the case when only places are added).

Conversely, assume that τ is a firing sequence of N which is not a firing sequence of $N[s]$ and assume that τ is a shortest such sequence. That is, $\tau = \tau' t$ such that τ' is a firing sequence both of N and of $N[s]$ leading to markings M in N (which enables t in N) and M' in $N[s]$ (which does not enable t in $N[s]$).

Because M' does not enable t in $N[s]$, there must be some place $q \in {}^\bullet t$ which is token-empty at M', and since by $M[t\rangle$ this is not true for any of the places of N, q must be one of the newly introduced places $q = s_\ell$. By the fact that $M'_0(q) = L_\ell$ and $M'(q) = 0$, transitions in $(q^\bullet \setminus {}^\bullet q)$ must have occurred L_ℓ times more often in τ' than transitions in $({}^\bullet q \setminus q^\bullet)$. But since t is also in $s^\bullet$ (but not in ${}^\bullet s$), τ is a sequence which is firable in N but contains transitions from $\mathit{supp}(J_\ell) \cap (s^\bullet \setminus {}^\bullet s)$ $L_\ell + 1$ times more often than transitions in $\mathit{supp}(J_\ell) \cap ({}^\bullet s \setminus s^\bullet)$, which contradicts the definition of L_ℓ as an asymmetric synchronic distance.

Thus, a firing sequence of N which is not also a firing sequence of $N[s]$ does not exist, and we have firing sequence equality, and hence also reachability graph isomorphism, between N and $N[s]$. The latter can be seen as follows. Let two nets $N, \widetilde{N}$ with the same transition set and the same set of firing sequences be given and construct a relation R between their respective sets of reachable markings by putting $(M, \widetilde{M}) \in R$ iff there is some τ with $M_0[\tau\rangle M$ and $\widetilde{M_0}[\tau\rangle\widetilde{M}$. Then R is surjective because any sequence is firable in $\widetilde{N}$ iff it is firable in N, and it is injective because the marking produced by a sequence is uniquely determined from the initial marking and the sequence itself.

Now we prove that the inequality $L_1 + \ldots + L_m \leq M_0(s)$ holds true. First, we will show that not only are there witnesses for the individual gaps $L_1, \ldots, L_m$, but there is even a witness realising all m gaps simultaneously.

Consider the first T-component, J_1, and consider any witness for L_1, that is, some sequence $M_0[\tau_1\rangle$ satisfying

$$\begin{array}{rcl} L_1 & = & \max\{\#(a, \tau_1) - \#(c, \tau_1) \\ & & \mid a \in \mathit{supp}(J_1) \cap (s^\bullet \setminus {}^\bullet s),\ c \in \mathit{supp}(J_1) \cap ({}^\bullet s \setminus s^\bullet)\}. \end{array} \quad (1)$$

Since τ_1 may contain transitions from other T-invariants $J_2, \ldots, J_m$, we will strive to 'remove' such transitions. Let $M_0[\tau\rangle M_0$ be a cycle whose Parikh vector is larger or equal to the Parikh vector of the sequence of non-J_1-transitions within τ_1. Such a cycle exists by Theorem 3. Since both τ_1 and τ are firable from M_0, Keller's theorem tells us that also $M_0[\tau\rangle M_0[(\tau_1 \stackrel{\bullet}{-} \tau)\rangle$. Let $\widetilde{\tau_1} = (\tau_1 \stackrel{\bullet}{-} \tau)$. By the fact that τ covers all non-J_1-transitions from τ_1 and because the T-invariants are transition-disjoint, $\widetilde{\tau_1}$ contains only transitions from J_1. Moreover, τ_1 is firable from M_0 and realises the gap L_1, since non-J_1-transitions do not contribute to formula (1) and $\widetilde{\tau_1}$ has exactly the same J_1-transitions as τ_1.

Repeating this procedure for all j from 2 to m, we find individual witnesses $\widetilde{\tau_1}, \ldots, \widetilde{\tau_m}$ for $L_1, \ldots, L_m$, respectively, such that every $\widetilde{\tau_j}$ contains transitions from J_j only. By Theorem 1 again, there is some sequence τ with $\Psi(\tau)(t) = \max_{1 \leq j \leq m} \Psi(\tau_j)(t)$ for all $t \in T$. Because all individual sequences are clean and because the J_ℓ are mutually transition-disjoint, the sequence τ realises all gaps $L_1, \ldots, L_m$ simultaneously.

Now we prove the desired inequality by contradiction. Assume otherwise, that is, assume that $L_1 + \ldots + L_m > M_0(s)$. By the above, we can find a witness τ realising all gaps $L_1, \ldots, L_m$ simultaneously. Since the sets $\{supp(J_\ell) \cap (s^\bullet \setminus {}^\bullet s) \mid 1 \leq \ell \leq m\}$ partition $s^\bullet \setminus {}^\bullet s$ and the sets $\{supp(J_\ell) \cap ({}^\bullet s \setminus s^\bullet) \mid 1 \leq \ell \leq m\}$ partition ${}^\bullet s \setminus s^\bullet$, this implies that transitions in $s^\bullet \setminus {}^\bullet s$ have occurred at least $M_0(s)+1$ times more often in τ than transitions in ${}^\bullet s \setminus s^\bullet$, creating a negative token count on s and leading to a contradiction.

Hence the assumption $L_1 + \ldots + L_m > M_0(s)$ is wrong and $L_1 + \ldots + L_m \leq M_0(s)$ is true instead. □ 5

According to the first part of Theorem 5, places $s_1, \ldots, s_m$ can be added to the net without altering its behaviour. According to the construction of the $s_1, \ldots, s_m$ and to the second part of Theorem 5, place s covers the sum of the places $s_1, \ldots, s_m$ in the sense that its F-connections are *exactly* the sum of the individual F'-connections of the s_ℓ and its initial marking is *equal to or larger* than the sum of the individual initial markings of the s_ℓ. Hence after $s_1, \ldots, s_m$ are added, s becomes a redundant place and can be omitted without altering the behaviour of the net. Altogether, we can replace place s by m places $s_1, \ldots, s_m$.

While the number of places properly increases by this transformation, the 'degree of conflict', that is, the number

$$\mathit{conf\text{-}deg} \;=\; \sum_{t \in T} |({}^\bullet t)^\bullet|$$

decreases. If Case 1 never arises before or during this construction, *conf-deg* will eventually be down to $|T|$, that is, we will get a net which is output-nonbranching and hence behaviourally conflict-free.

Note that Theorem 5 is also true if $|\mathcal{J}_s| = 1$, but does not, in this case, lead to a reduction of *conf-deg*.

To sum this section up, if a place s is affected by m simple cycles, then it can be split into m places, each of which is 'responsible' for one of the cycles.

5 Branching places surrounded by a single simple cycle

In Case 1, when there is a single simple cycle through all transitions bordering on a place s with two or more output transitions, a similar analysis and reduction may not necessarily be possible. For instance, consider the net shown in Figure 3. The transition inscriptions denote their values in the only minimal realisable T-invariant, defined by the transition counts on one of the cycles of the reachability graph. In state M', the output transitions of place s are concurrently enabled. We might consider the subset $A = \{t_6\}$ of $s^\bullet$ and the subset $B = \{t_5\}$ of ${}^\bullet s$ because t_5 and t_6 occur equally often in a cycle. Note that both $asd_{M_0}(A, B)$ and $asd_{M_0}(B, A)$ are finite. Moreover, for the complementary sets $A' = s^\bullet \setminus A = \{t_7\}$ and $B' = {}^\bullet s \setminus B = \{t_3, t_4\}$, the values of $asd_{M_0}(A', B')$ and $asd_{M_0}(B', A')$, as well as the other combinations, $asd_{M_0}(A, B')$, $asd_{M_0}(B', A)$, $asd_{M_0}(A', B)$ and $asd_{M_0}(B, A')$, are finite. We can thus add (up to eight) places reflecting these finite synchronic distances without changing the behaviour of the net. Nevertheless, even after addition of all these places, the place s cannot be removed without changing behaviour, because the sequence $t_1 t_5 t_6 t_8 t_{11} t_2 t_7$ is firable in the net so obtained. As can be seen from the reachability graph in Figure 3, this sequence is not firable in the original net.

The same net proves that, in a persistent Petri net, a forward branching place can have both options (a) and (b) (see section 2). There are reachable markings that enable only one of the output transitions of place s (namely, the markings reached after the occurrences of $t_1 t_5$ and after $t_2 t_5$, respectively), and there is a reachable marking (reached after the occurrence of t_4) that enables both output transitions concurrently.

The *asd* approach fails for another reason in the example shown in Figure 4. The only realisable (minimal and semipositive) T-invariant assigns *1* to transition c, *3* to transition d and *2* each to transitions a and b. More concretely, every simple cycle is a permutation of $adzbc'dxadybc$. Therefore, we do not find two proper nonempty subsets $A \subseteq s^\bullet$ and $B \subseteq {}^\bullet s$ such that transitions in A and transitions in B occur equally often in a cycle. For example, $asd_{M_0}(a, c)$ is infinite.

6 Concluding remarks

This paper is part of a longer-term goal, namely to prove that bounded, reversible and persistent Petri nets are separable. Separability can be described informally as follows. Let k denote the greatest common divisor of the numbers $M_0(s)$, for places s and initial marking M_0 of some net. Then the net is (k-)separable if it behaves as k independent copies of the net arising when M_0 is replaced by M_0' with $M_0'(s) = M_0(s)/k$, for every place s.

We can presently imagine two possible ways of proving this conjecture. One possibility is a direct proof. Another possibility is using the results in [3, 4], where separability has been proved for live, bounded, and behaviourally conflict-free nets. In order to be able to make use of the second avenue, a transformation from persistent

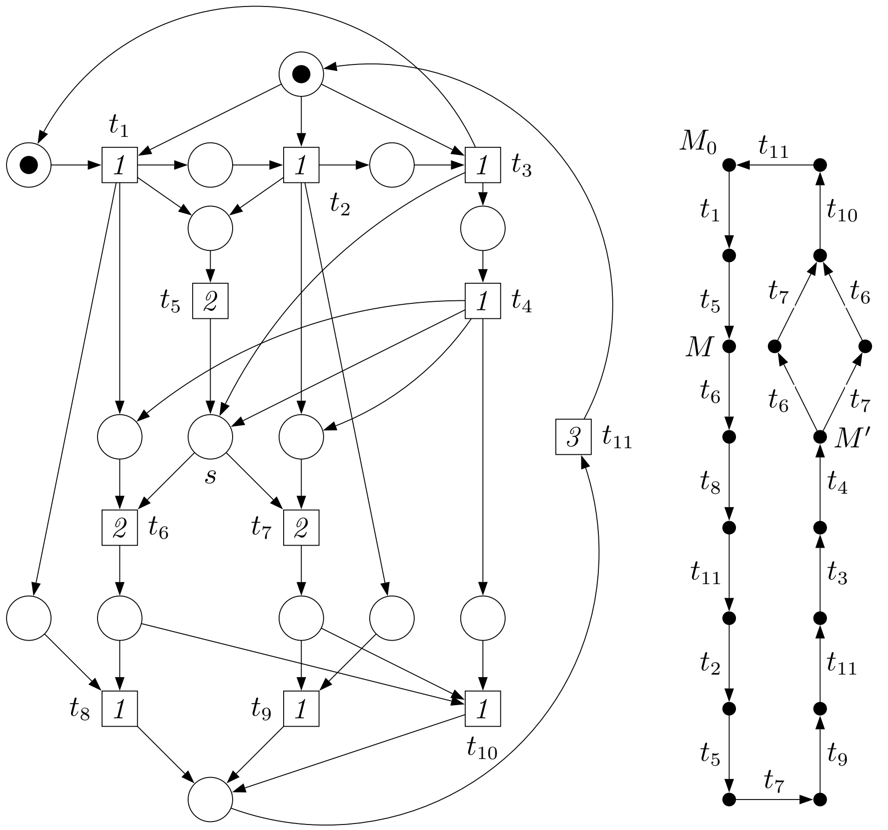

Figure 3: A third persistent Petri net and its reachability graph

to behaviourally conflict-free nets such as explored in this paper is needed.

Independently of this connection, we are also looking at ways of adapting the construction by Ramamoorthy et al. [11] (mentioned also in Murata [10]) to yield transition-labelled bfc nets which are bisimilar [9] to a given persistent net.

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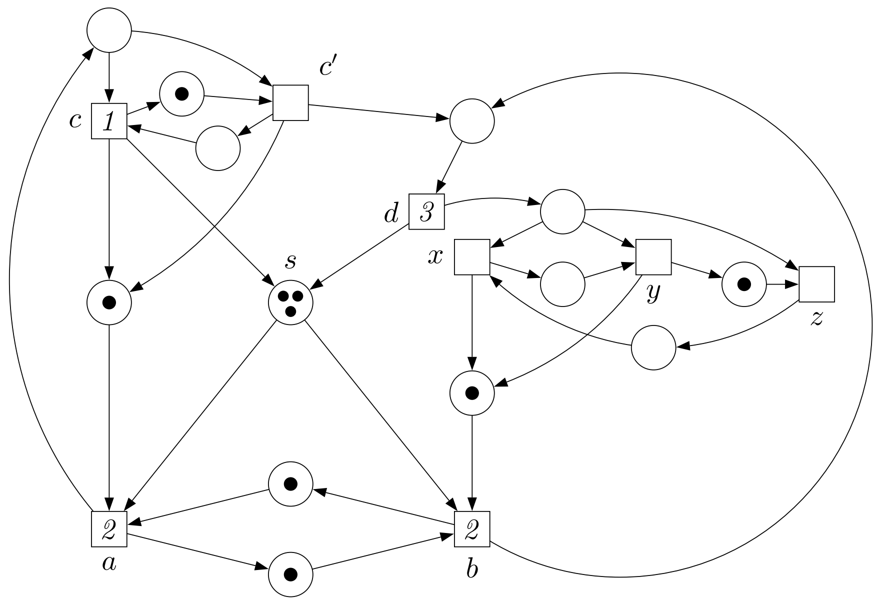

Figure 4: A fourth persistent Petri net

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