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Vicinity Respecting Homomorphisms for Abstracting System Requirements

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Abstract. This paper is concerned with the problem of structuring system and software requirements on an abstract conceptual level. Channel/Agency Petri nets are taken as a formal model. They allow to represent functional aspects as well as data aspects of the requirements in a graphical way. Vicinity respecting homomorphisms are presented as a means to refine and abstract these nets. They preserve paths, i.e., dependencies between computational elements. Further, it is shown that they preserve important structural properties of nets, such as S- and T-components, traps and siphons. These structural objects are helpful to gain better understanding of the whole system. For example, S-components give the main streams of choice while T-components give the main streams of concurrency. Moreover, these object have important interpretations for marked Petri nets and can therefore be used for the analysis of system models at more concrete levels.

0 Introduction

A nontrivial task in the design of large and complex systems is to organize the requirements into a coherent structure. Usually, this organization is a gradual process which involves refinement and abstraction between different conceptual levels of the system. In this paper we take Channel/Agency Petri nets [22] to model systems and propose vicinity respecting homomorphisms as a means to refine and abstract these nets.

Petri Nets (see e.g. [21]) are special bipartite graphs. The two different types of nodes are called *places* and *transitions*. Places are considered as passive elements that allow to represent data while transitions are considered as active elements that allow to model activities or functions of system components. Directed edges between places and transitions reflect the flow and transformation of data and control. Thus a path formed by consecutive edges represents causal dependencies between its first and its last element. With these two types of elements, Petri nets combine a data oriented view with a process oriented view of the system.

For large and complex systems it is necessary to have models for different levels of abstraction and notions for refinement and abstraction to formulate relations between these models. Channel/Agency Petri nets are a Petri net model where all elements of a net are labeled by informal descriptions. They have been proposed for the conceptual modeling of the architecture of information systems e.g. in [5,6,14]. As shown in [22] they can be used for different levels of abstraction. On a low level of abstraction containing all details nets can be equipped with markings and a notion of behaviour which simulates the behaviour of the modeled system. In this way Petri nets can be used as a means for prototyping.

We introduce vicinity respecting homomorphisms of Petri nets to formalize the refinement and abstraction relations between nets. This encompasses modular techniques because each composition of subsystems may be viewed as an identification of the respective interface elements and thus as a particular abstraction. Vicinity respecting homomorphisms rely on the graph structure of a net. They are special graph homomorphisms that are able to formalize abstractions including contractions of graphs not only in their breadth but also in their length. Working with graphs or nets, we distinguish the pre- and the post-vicinity of an element x: the pre-vicinity ${}^{\odot}x$ includes the pre-set of x (the set of elements with an arc leading to x) and x itself while the post-vicinity $x^{\odot}$ includes the post-set together with x. In vicinity-respecting homomorphisms it is required that the pre-vicinity of an element is mapped

either surjectively onto the pre-vicinity of its image *or* entirely to its image and likewise for post-vicinities.

Figure 1 shows four models of a sender/receiver system, representing four levels of abstraction. As usual, places are represented by circles and transitions are represented by squares. The interrelation between these models is formally given by mappings between the respective elements. For example, the place *message in channel* in the second model is refined to the place *message in channel A* and to the subsystem containing the place *message in channel B start*, the transition *message transmission* and the place *message in channel B end* in the first model. These mappings are homomorphisms between the nets.

The definition of vicinity respecting homomorphisms is based on the local vicinities of elements. This concept suffices to preserve important global structural properties like connectedness. If two elements of a net are connected by a path then the respective system components are in a causal dependency relation. Because they preserve paths, vicinity respecting homomorphisms not only respect dependency but also its complementary relation independency.

Petri nets not only allow to combine data- and function-oriented views of a system. They also allow to concentrate on either aspect. The data aspect including nondeterministic choice is reflected by S-components, i.e. by subnets where every transition has at most one input place and at most one output place, i.e. where only places are branched. For example, in the first net of Figure 1 we have an S-component for the sender containing the places *idle* and *message produced* and the transitions *produce message*, *cancel message*, *send message in channel A* and *send message in channel B*. Similarly, the receiver is represented by an S-component. Likewise, T-components represent an activity-oriented view, where only transitions are branched. Petri nets that are covered by S- and T-components allow for a compositional interpretation of these two aspects. We show that vicinity respecting net homomorphisms preserve coverings by S- and T-components. As a consequence, they respect the notions of choice (a forward branching place) and of synchronization (a backward branching transition).

This paper gathers, generalizes and deepens results obtained in [8,17]. In section 1 we investigate homomorphisms of arbitrary graphs. Amongst other results it is shown that for arbitrary surjective graph homomorphisms the image of a path of the source graph is a path of the target graph while the converse direction – a path of the target graph is necessarily the image of some path of the source graph – does not necessarily hold but holds for vicinity respecting homomorphisms. Section 2 is a section which introduces Petri nets and transfers the notion of vicinity respecting homomorphisms to them. In

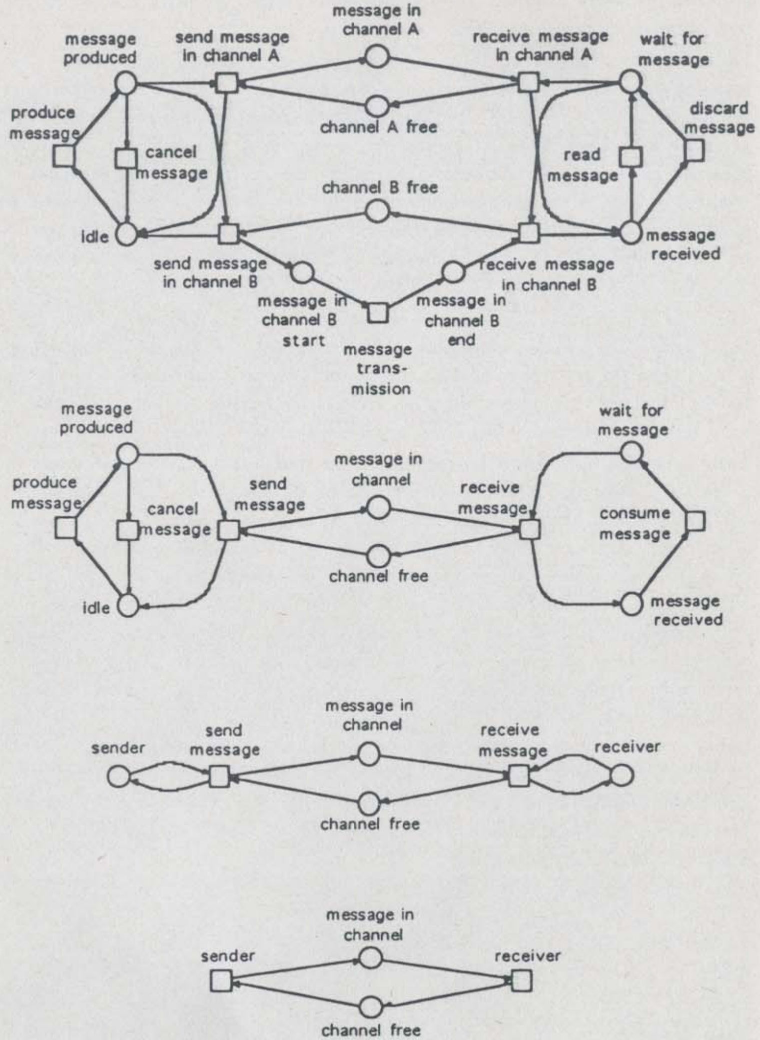

Figure 1: *Four levels of abstraction of a sender/receiver model*

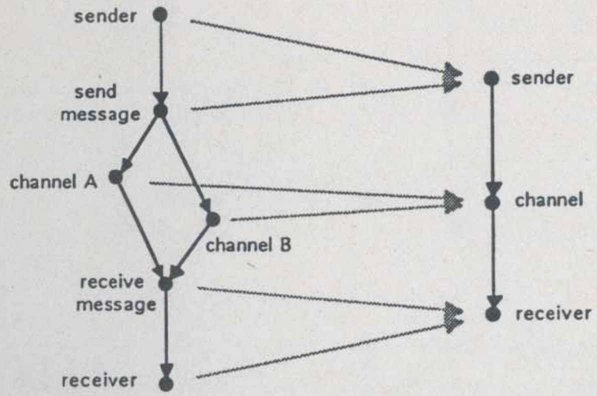

Figure 2: *Two graphs modeling a sender/receiver system*

section 3 we show that vicinity respecting homomorphisms respect coverings by S- and T-components of Petri nets and draw consequences for Petri nets composition. Siphons and traps are concepts known from Petri net theory that allow for an analysis of the data contained in sets of places. Section 4 proves that vicinity respecting homomorphisms preserve siphons and traps. Finally, section 5 concludes this paper with final remarks and related works.

1 Graph Homomorphisms

Petri nets are special graphs. Vicinity respecting homomorphisms will be defined for arbitrary graphs in this section.

Figure 2 shows a model of a sender/receiver system on the left hand side and a coarser view of the same system on the right hand side. The left model can be viewed as a refinement of the right model. The interrelation between the graphs is given by a mapping, depicted by arrows between the vertices of the graphs. This mapping is a particular graph homomorphism. As we shall see, in this example dependencies between vertices of the source graph are strongly related to dependencies between vertices of the target graph.

We start with a formal introduction of graphs and related concepts. We consider only finite directed graphs without multiple edges and without loops.

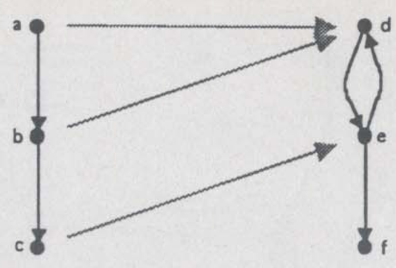

Figure 3: *A graph homomorphism*

Definition 1.1

A *graph* is a pair (X, F) where X is a finite set and $F \subseteq X \times X$. The set X is the set of *vertices* and F is the set of *edges* of the graph. A *loop* is an edge (x, x) where $x \in X$. The graph is said to be *loop-free* if no edge is a loop.

The classical notion of graph homomorphism [19] respects edges in the sense that the images of connected vertices are again connected. Since we also consider contractions of loop-free graphs, where two connected vertices are mapped to one vertex without a loop, a slightly more liberal definition will be employed; we allow the images of connected vertices to be either connected or identical.

Definition 1.2

Let (X, F) and (X', F') be graphs. A mapping $\varphi: X \to X'$ is a *graph homomorphism* (denoted by $\varphi: (X, F) \to (X', F')$) if, for every edge $(x, y) \in F$, either there is an edge $(\varphi(x), \varphi(y)) \in F'$ or $\varphi(x) = \varphi(y)$.

To describe the environment of an element we shall use the notions of pre- and post-sets and related notions of pre- and post-vicinities.

Notation 1.3

Given a graph (X, F), we denote by ${}^\bullet x$ the *pre-set* of a vertex $x \in X$, which is defined by ${}^\bullet x = \{y \in X \mid (y, x) \in F\}$.

Similarly, the *post-set* $x^\bullet$ is defined by $x^\bullet = \{y \in X \mid (x, y) \in F\}$.

The *pre-vicinity* of x is defined by ${}^{\odot}x = \{x\} \cup {}^\bullet x$,

the *post-vicinity* of x is defined by $x^{\odot} = \{x\} \cup x^\bullet$.

In Figure 3, ${}^\bullet a = \emptyset$, ${}^{\odot}a = \{a\}$ and $a^{\odot} = \{a, b\}$. It is immediate to note that ${}^{\odot}x = {}^\bullet x$ if and only if there is a loop $(x, x) \in F$. Using the notion of vicinities, homomorphisms can be phrased as: the image of the pre-vicinity is included in the pre-vicinity of the image and the image of the post-vicinity is included in the post-vicinity of the image.

Proposition 1.4

Let (X, F) and (X', F') be graphs. A mapping $\varphi: X \to X'$ is a graph homomorphism if and only if, for all $x \in X$, we have $\varphi({}^{\odot}x) \subseteq {}^{\odot}(\varphi(x))$ and $\varphi(x^{\odot}) \subseteq (\varphi(x))^{\odot}$. ∎

Note that $\varphi({}^\bullet x) \subseteq {}^\bullet(\varphi(x))$ does not hold for arbitrary graph homomorphisms because in case of contractions elements of the pre-set of a vertex x can be mapped to $\varphi(x)$. The same holds for post-sets.

Definition 1.5

A sequence $x_1\, x_2 \ldots x_n$ $(n \geq 1)$ of vertices of a graph is a *path* if there exist edges $(x_1, x_2), \ldots, (x_{n-1}, x_n)$ of the graph.

A graph is *strongly connected* if for any two vertices x and y there exists a path $x \ldots y$.

We allow a single element to be a path as a technical facility. Since consecutive vertices of a graph can be mapped onto a single element without a loop the sequence of images of some path elements is not necessarily a path of the target graph. So we define for loop-free graphs the image of a path to ignore stuttering of vertices.

Definition 1.6

Let (X, F) and (X', F') be loop-free graphs and let $\varphi: (X, F) \to (X', F')$ be a graph homomorphism. The *image of a path* $x_1 \ldots x_n$ of (X, F) is inductively defined by

$$\varphi(x_1 \ldots x_m) = \begin{cases} \varphi(x_1) & \text{if } m = 1 \\ \varphi(x_1 \ldots x_{m-1}) & \text{if } m > 1 \text{ and } \varphi(x_{m-1}) = \varphi(x_m) \\ \varphi(x_1 \ldots x_{m-1})\, \varphi(x_m) & \text{if } m > 1 \text{ and } \varphi(x_{m-1}) \neq \varphi(x_m) \end{cases}$$

Using Definition 1.6, the image of the path abc in Figure 3 is de. Graph homomorphisms do not preserve edges but they preserve paths between vertices, as shown in the next lemma.

Lemma 1.7

Let (X, F) and (X', F') be loop-free graphs and let $\varphi: (X, F) \rightarrow (X', F')$ be a graph homomorphism. If $x_1 \ldots x_n$ is a path of (X, F) then $\varphi(x_1 \ldots x_n)$ is a path of (X', F') leading from $\varphi(x_1)$ to $\varphi(x_n)$.

Proof: We proceed by induction on n.

If $n = 1$ then $\varphi(x_1)$ is a path of (X', F').

Let $n > 1$ and assume that $\varphi(x_1 \ldots x_{n-1})$ is a path leading from $\varphi(x_1)$ to $\varphi(x_{n-1})$. We have $(x_{n-1}, x_n) \in F$ by the definition of a path. By the homomorphism property, we can distinguish two cases:

1. $(\varphi(x_{n-1}), \varphi(x_n)) \in F'$. Then $\varphi(x_1 \ldots x_n) = \varphi(x_1 \ldots x_{n-1})\, \varphi(x_n)$ is a path of (X', F') leading from $\varphi(x_1)$ to $\varphi(x_n)$
2. $\varphi(x_{n-1}) = \varphi(x_n)$. Then $\varphi(x_1 \ldots x_n) = \varphi(x_1 \ldots x_{n-1})$. By assumption, this is a path leading from $\varphi(x_1)$ to $\varphi(x_{n-1})$. Since $\varphi(x_{n-1}) = \varphi(x_n)$ this path leads to $\varphi(x_n)$. ■

Surjectivity is a first condition when graph homomorphisms are used for abstractions. The following corollary states that surjective graph homomorphisms preserve strong connectivity of graphs.

Corollary 1.8

Let (X, F) and (X', F') be loop-free graphs and let $\varphi: (X, F) \rightarrow (X', F')$ be a surjective graph homomorphism. If (X, F) is strongly connected then (X', F') is also strongly connected.

Proof: Let $x', y' \in X'$. Since φ is surjective there are $x, y \in X$ such that $\varphi(x) = x'$ and $\varphi(y) = y'$. There is a path leading from x to y because (X, F) is strongly connected. Using Lemma 1.7, some path of (X', F') leads from x' to y'. ■

Surjectivity concerns vertices only. An additional requirement is that every edge of a target graph reflects a connection between respective vertices of the source graph. We call such a homomorphism a quotient.

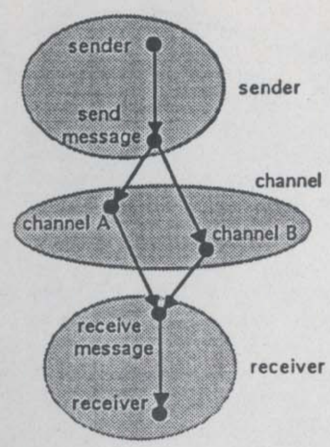

Figure 4: *Another representation of the sender/receiver model*

Definition 1.9

Let (X, F) and (X', F') be loop-free graphs. A graph homomorphism $\varphi: (X, F) \rightarrow (X', F')$ is called *quotient* if

1. the mapping $\varphi: X \rightarrow X'$ is surjective, and
2. for every edge $(x', y') \in F'$ there exists an edge $(x, y) \in F$ such that $\varphi(x) = x'$ and $\varphi(y) = y'$.

The name "quotient" is justified because for quotients, target graphs are determined up to renaming by the equivalence classes of vertices that are mapped onto the same vertex (see [9]). Therefore, we can represent quotients graphically by solely depicting equivalence classes as shown in Figure 4. The edges of the target graph are the equivalence classes of the edges of the source graph. There is a vertex connecting two edges of the target graph if and only if there is at least one edge connecting elements of the respective sets of vertices in the source graph.

When thinking of (X', F') as of an abstraction of (X, F), dependencies between nodes of X', that are represented through paths, have to mirror dependencies already present in X. Therefore, since dependencies between vertices are modeled by paths connecting the vertices, we look for a converse of Lemma 1.7. For quotients, this lemma has a weak converse: every path of the

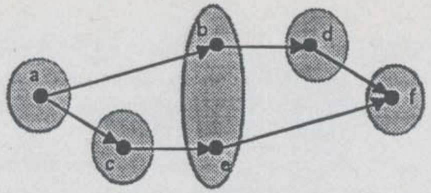

Figure 5: *A path of the target graph is not necessarily the image of a path of the source graph*

target graph with at most two vertices is the image of a path of the source graph. The same does not necessarily hold for longer paths, as shown in Figure 5. The target graph has a path $\varphi(a)\,\varphi(b)\,\varphi(f)$ which is not the image of a path of the source graph. What is wrong with this homomorphism? The post-vicinity of b is the set $\{b, d\}$. The post-vicinity of the image of b contains three vertices, namely $\varphi(b), \varphi(d)$ and $\varphi(f)$. So the image of the post-vicinity of b is properly included in the post-vicinity of the image. We say that the post-vicinity is not respected.

We strengthen Proposition 1.4 and define homomorphisms that respect vicinities of vertices. Since we still allow contractions we also have to consider 'inner' vertices where the entire post-vicinity of a vertex is mapped to one element, i.e. to the image of the vertex.

Definition 1.10

Let (X, F) and (X', F') be loop-free graphs.

A graph homomorphism $\varphi: (X, F) \to (X', F')$ is called *pre-vicinity respecting* if, for every $x \in X$, either $\varphi(^{\odot}x) = {}^{\odot}(\varphi(x))$ or $\varphi(^{\odot}x) = \{\varphi(x)\}$.

Similarly, φ is called *post-vicinity respecting* if, for every $x \in X$, either $\varphi(x^{\odot}) = (\varphi(x))^{\odot}$ or $\varphi(x^{\odot}) = \{\varphi(x)\}$.

A graph homomorphism is called *vicinity respecting* if it is pre-vicinity respecting and post-vicinity respecting.

In the next theorem we show that for surjective post-vicinity respecting homomorphisms of strongly connected graphs there is a converse of Lemma 1.7. By symmetry, the same holds for pre-vicinity respecting homomorphisms.

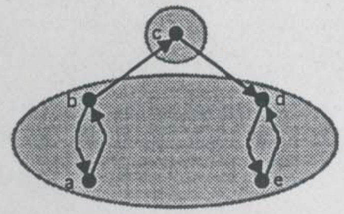

Figure 6: *For non-strongly connected graphs, a path of the target graph by a vicinity respecting quotient is not necessarily the image of a path*

Theorem 1.11

Let (X, F) and (X', F') be loop-free graphs such that (X, F) is strongly connected, and let $\varphi: (X, F) \to (X', F')$ be a surjective post-vicinity respecting graph homomorphism. Let $x'_1 \dots x'_m$ be a path of (X', F') such that for all i, $1 \le i < m$, $x'_i \ne x'_{i+1}$ (no stuttering). Then there is a path $x_1 \dots x_n$ of (X, F) such that $\varphi(x_1 \dots x_n) = x'_1 \dots x'_m$.

Proof: We proceed by induction on m.

Let $m = 1$. Since φ is surjective, some $x_1 \in X$ satisfies $\varphi(x_1) = x'_1$. The path consisting of x_1 satisfies the required property.

Let $m > 1$. Assume that $x_1 \dots x_k$ is a path of (X, F) such that $\varphi(x_1 \dots x_k) = x'_1 \dots x'_{m-1}$. Since φ is surjective, some $y \in X$ satisfies $\varphi(y) = x'_m$. Since the graph (X, F) is strongly connected, it contains a path leading from x_k to y. Consider the first vertex x_{i+1} in this path that is not mapped to x'_{m-1}. Such a vertex exists because the last vertex y in the path is mapped to x'_m and $x'_{m-1} \ne x'_m$ by the assumption. By definition, the predecessor x_i of x_{i+1} is mapped to x'_{m-1} and so are all vertices in the path $x_k \dots x_i$.

We have $x_{i+1} \in x_i^{\odot}$. Since $\varphi(x_{i+1}) \ne \varphi(x_i)$ we obtain $\varphi(x_i^{\odot}) \ne \{\varphi(x_i)\}$. Using the definition of a post-vicinity respecting homomorphism we conclude that $\varphi(x_i^{\odot}) = (\varphi(x_i))^{\odot}$. Therefore, since $x'_m \in (x'_{m-1})^{\odot} = (\varphi(x_i))^{\odot}$, some vertex $z \in x_i^{\odot}$ is mapped to x'_m. This vertex z cannot be x_i itself, hence it is in $x_i^{\bullet}$.

Now the path $x_1 \dots x_k \dots x_i z$ combined from the paths $x_1 \dots x_k$, $x_k \dots x_i$ and $x_i z$ is mapped to $x'_1 \dots x'_m$, which concludes the proof. ■

The example in Figure 6 shows that in the previous theorem it is necessary that the source graph (X, F) is strongly connected. This graph homomorphism φ

is a vicinity respecting quotient. The target graph has a path $\varphi(c)\,\varphi(d)\,\varphi(c)$ which is not the image of a path of the source graph. There exist simpler examples to demonstrate that not all vicinity respecting homomorphisms satisfy a converse of Lemma 1.7. It is easy to find an example where the source graph has only two edges. We have chosen a slightly more involved example to demonstrate that neither requiring weak connectedness nor nonempty pre- or post-sets for all vertices constitute sufficient conditions.

Corollary 1.12

Let (X, F) and (X', F') be loop-free graphs such that (X, F) is strongly connected and let $\varphi: (X, F) \to (X', F')$ be a surjective and post-vicinity respecting graph homomorphism. Then φ is a quotient.

Proof: If $|X'| \le 1$, then $F' = \emptyset$ and we are finished. So assume $|X'| > 1$.

Let $(x', y') \in F'$. By Theorem 1.11 there exists a path $x_1 \dots x_n$ of X with $\varphi(x_1 \dots x_n) = (x', y')$. Let x_i be the last element of the path with $\varphi(x_i) = x'$. Then $(x_i, x_{i+1}) \in F$ and $\varphi(x_{i+1}) = y'$, which was to prove. ∎

The following result, that will be used later, is much weaker than 1.12 but it holds for arbitrary surjective mappings.

Lemma 1.13

Let (X, F) and (X', F') be loop-free graphs such that (X, F) is strongly connected and $|X'| > 1$. If $\varphi: (X, F) \to (X', F')$ is a surjective mapping then we have for every $x' \in X'$:

1. *there exists an arc $(x, y) \in F$ with $\varphi(x) = x'$ and $\varphi(y) \neq x'$;*
2. *there exists an arc $(x, y) \in F$ with $\varphi(y) = x'$ and $\varphi(x) \neq x'$.*

Proof: We show only the first part, the second one being similar.

Let y' be an element of X' distinct from x' (which is possible because $|X'| > 1$). Since φ is surjective there are $c, d \in X$ with $\varphi(c) = x'$ and $\varphi(d) = y'$. Since (X, F) is strongly connected, there exists a path $x_1 \dots x_m$ of X with $x_1 = c$ and $x_m = d$. Let i be the least index such that $\varphi(x_i) = x'$ and $\varphi(x_{i+1}) \neq x'$. With $(x, y) = (x_i, x_{i+1})$ we are finished. ∎

Concentrating on different elements which are mapped onto the same image instead of comparing source graph and target graph leads to another aspect of vicinity respecting homomorphisms in the case of quotients.

Proposition 1.14

Let (X, F) and (X', F') be loop-free graphs and let $\varphi: (X, F) \to (X', F')$ be a quotient. φ is vicinity respecting if and only if for all $x, y \in X$ satisfying $\varphi(x) = \varphi(y)$:

1. $\varphi({}^{\odot}x) = \{\varphi(x)\}$ *or* $\varphi({}^{\odot}y) = \{\varphi(y)\}$ *or* $\varphi({}^{\odot}x) = \varphi({}^{\odot}y)$;
2. $\varphi(x^{\odot}) = \{\varphi(x)\}$ *or* $\varphi(y^{\odot}) = \{\varphi(y)\}$ *or* $\varphi(x^{\odot}) = \varphi(y^{\odot})$.

Proof: It is immediate that Definition 1.10 implies *1.* and *2.* We only show that *1.* implies pre-vicinity respecting; showing that *2.* implies post-vicinity respecting is similar.

Let $x \in X$ such that $\varphi({}^{\odot}x) \neq \{\varphi(x)\}$.

Let $z' \in {}^{\bullet}\varphi(x)$. Then there exist $y, z \in X$ such that $z \in {}^{\bullet}y$, $\varphi(z) = z'$ and $\varphi(y) = \varphi(x)$ because φ is a quotient. By *1.* we obtain that $\varphi({}^{\odot}x) = \varphi({}^{\odot}y)$. Since $z \in {}^{\odot}y$, some element in ${}^{\odot}x$ is mapped to z'.

Since z' was chosen arbitrarily in ${}^{\bullet}\varphi(x)$ we finally obtain $\varphi({}^{\odot}x) = {}^{\odot}\varphi(x)$. ∎

In the proof of Proposition 1.14, we showed that for any element $z' \in {}^{\bullet}\varphi(x)$ there exists an element $z \in {}^{\bullet}x$ with $\varphi(z) = z'$. From this fact, we deduce immediately the following technical corollary that will be used later.

Corollary 1.15

Let (X, F) and (X', F') be loop-free graphs and let $\varphi: (X, F) \to (X, F')$ be a vicinity respecting quotient. Then we have for every $x \in X$:

1. *if* $\varphi({}^{\odot}x) \neq \{\varphi(x)\}$ *then* $|{}^{\bullet}\varphi(x)| \le |{}^{\bullet}x|$;
2. *if* $\varphi(x^{\odot}) \neq \{\varphi(x)\}$ *then* $|\varphi(x)^{\bullet}| \le |x^{\bullet}|$. ∎

2 Net homomorphisms

A net can be seen as a loop-free graph (X, F) where the set X of vertices is partitioned into a set S of places and a set T of transitions such that F may not relate two places or two transitions. Formally:

Definition 2.1

A triple $N = (S, T; F)$ is called a *Petri net* or a *net* if:

1. S and T are disjoint sets;
2. $F \subseteq (S \times T) \cup (T \times S)$.

The set $X = S \cup T$ is the set of *elements* or *nodes* of the net.

This definition allows to consider nets with isolated elements, i.e. elements with empty pre- and post-sets, in contrast to the definition given in [2] (a net, as defined here, is called *pre-net* in [2]). Graphically *places* are represented by circles, *transitions* are represented by squares and the *flow relation* F is represented by arrows between the elements. Note that we do not consider markings and behavioural notions but concentrate on the structure of net models.

We use the following convention: indices and primes used to denote a net N are carried over to all parts of N. For example, speaking of a net N_i', we implicitly understand $N_i' = (S_i', T_i'; F_i')$ and $X_i' = S_i' \cup T_i'$.

A consequence of Definition 2.1 is that the pre-set and the post-set of a place are sets of transitions, and the pre-set and the post-set of a transition are sets of places.

The transitions of a net model the active subsystems, i.e. functions, operators, transformators etc. They are only connected to places which model passive subsystems, i.e. data, messages, conditions etc. On a conceptual level, it is not always obvious to classify a subsystem active or passive. The decision to model it by a place or by a transition is based on the interaction of the subsystem with its vicinity. As an example, consider a *channel* that is connected to functional units that *send* and *receive* data through the channel. Then the channel has to be modeled by a place. In contrast, if the channel is connected to data that have to be sent on one side and to already received data on the other side then the channel is modeled by a transition. As we shall see, a transition may represent a subsystem that is modeled by a net containing places and transitions on a finer level of abstraction. The same holds respectively for places.

An arrow in a Petri net either leads from a place to a transition or from a transition to a place. In the first case the place is interpreted as a pre-requisite (pre-condition, input) for the transition which can be consumed by the action modeled by the transition. In the second case the place is interpreted as a post-requisite (post-condition, output) for the transition which can be produced by

the action modeled by the transition. In this sense, arrows are used to denote two different types of relations between the elements of a Petri net.

homomorphisms of Petri nets are particular graph homomorphisms that additionally respect the type of relation between the elements given by arrows. Since we again allow contractions, places can be mapped to transitions and transitions can be mapped to places. However, if two connected elements are not mapped to the same element of the target net, then the place of the two has to be mapped to a place and the transition has to be mapped to a transition.

So Definition 1.2 becomes for Petri nets:

Definition 2.2

Let N, N' be nets. A mapping $\varphi: X \to X'$ is called *net homomorphism*, denoted by $\varphi: N \to N'$, if for every edge $(x, y) \in F$ holds:

1. if $(x, y) \in F \cap (S \times T)$ then either $(\varphi(x), \varphi(y)) \in F' \cap (S' \times T')$ or $\varphi(x) = \varphi(y)$ and
2. if $(x, y) \in F \cap (T \times S)$ then either $(\varphi(x), \varphi(y)) \in F' \cap (T' \times S')$ or $\varphi(x) = \varphi(y)$.

This definition is equivalent to the one given in [12] or [11]. Note that there also are similar but slightly different notions in the literature, see e.g. [1]. A consequence of our definition is that a transition is allowed to be mapped to a place only if all elements of its pre-vicinity and post-vicinity are mapped to the same place, and vice versa.

Lemma 2.3

Let $\varphi: N \to N'$ be a net homomorphism. Then:

1. *if a transition $t \in T$ is mapped to a place s' then $\varphi({}^{\circledcirc}t \cup t^{\circledcirc}) = \{s'\}$;*
2. *if a place $s \in S$ is mapped to a transition t' then $\varphi({}^{\circledcirc}s \cup s^{\circledcirc}) = \{t'\}$.*

Proof: We only show part *1*, part *2* being similar.

Let $t \in T$, $s' \in S'$ such that $\varphi(t) = s'$. Then for each place $s \in {}^{\bullet}t$ we have $(s, t) \in F \cap S \times T$ and $(\varphi(s), \varphi(t)) \notin S' \times T'$ and therefore $\varphi(s) = \varphi(t)$. Likewise, each place $s \in t^{\bullet}$ satisfies $\varphi(s) = \varphi(t)$.

Since $\varphi({}^{\circledcirc}t \cup t^{\circledcirc}) = \varphi({}^{\bullet}t) \cup \varphi(t) \cup \varphi(t^{\bullet})$ and $\varphi(t) = s$ we obtain the result. ■

Corollary 2.4

Let $\varphi: N \to N'$ be a net homomorphism and let $(x,y) \in F$ such that $\varphi(x) \neq \varphi(y)$. Then $\varphi(x) \in S'$ if and only if $x \in S$ and $\varphi(y) \in S'$ if and only if $y \in S$. ■

For Petri nets the vicinity respecting homomorphism definition can be split into two notions: homomorphisms that respect the vicinity of places and homomorphisms that respect the vicinity of transitions.

Definition 2.5

Let $\varphi: N \to N'$ be a net homomorphism.

1. φ is *S-vicinity respecting* if, for every $x \in S$:
   (a) $\varphi(^{\odot}x) = {}^{\odot}(\varphi(x))$ or $\varphi(^{\odot}x) = \{\varphi(x)\}$ and
   (b) $\varphi(x^{\odot}) = (\varphi(x))^{\odot}$ or $\varphi(x^{\odot}) = \{\varphi(x)\}$.
2. φ is *T-vicinity respecting* if, for every $x \in T$:
   (a) $\varphi(^{\odot}x) = {}^{\odot}(\varphi(x))$ or $\varphi(^{\odot}x) = \{\varphi(x)\}$ and
   (b) $\varphi(x^{\odot}) = (\varphi(x))^{\odot}$ or $\varphi(x^{\odot}) = \{\varphi(x)\}$.
3. φ is *vicinity respecting* if it is both S-vicinity respecting and T-vicinity respecting.

A subnet of a net is generated by its elements and preserves the flow relation between its elements. We will be interested in subnets that are connected to the remaining part only via places or only via transitions.

Notation 2.6

The pre-set (post-set) notions of 1.3 are generalized to sets $A \subseteq X$ of elements

$$^{\bullet}A = \bigcup_{x\in A} {}^{\bullet}x, \quad A^{\bullet} = \bigcup_{x\in A} x^{\bullet}$$

Definition 2.7

Let N be a net. $X_1 \subseteq X$ generates the *subnet*

$$N_1 = (S \cap X_1, T \cap X_1; F \cap (X_1 \times X_1)).$$

N_1 is called transition-bordered if $^{\bullet}S_1 \cup S_1^{\bullet} \subseteq T_1$.
N_1 is called place-bordered if $^{\bullet}T_1 \cup T_1^{\bullet} \subseteq S_1$.

A single transition of a net constitutes a transition-bordered subnet. Similarly, a place constitutes a place-bordered subnet. Figure 7 shows three nets. The net in the middle of the figure is a subnet of the net on the left hand side. This subnet is generated by its set of elements $\{a,b,c,d,f,h,i,m,o\}$. It is transition-bordered because, for its places a, d, f and m, it contains all transitions in the pre- and the post-set. Similarly, the net on the right hand side is place-bordered.

Net homomorphisms allow to map places to transitions and vice versa. Nevertheless, the role of 'active' and 'passive' components of a net are preserved in the following sense. The refinement of a transition-bordered subnet is a transition-bordered subnet, i.e., the reverse image of the elements of a transition-bordered subnet generates a transition-bordered subnet of the source net. Similarly, the set of elements of the source net that are mapped to some place-bordered subnet of the target net constitute a place-bordered subnet of the source net. The following results have been proved in [10] in a topological framework.

Proposition 2.8

Let $\varphi: N \to N'$ be a net homomorphism.

1. *If N'_1 is a transition-bordered subnet of N' then $\{x \in X \mid \varphi(x) \in X'_1\}$ generates a transition-bordered subnet of N.*
2. *If N'_1 is a place-bordered subnet of N' then $\{x \in X \mid \varphi(x) \in X'_1\}$ generates a place-bordered subnet of N.*

Proof: We show only part *1*, part *2* being similar.
Let $(x,y) \in F$. Assume that $\varphi(x) \notin N'_1$ and $\varphi(y) \in N'_1$. Since N'_1 is a transition-bordered subnet of N', $\varphi(x)$ is a place and $\varphi(y)$ is a transition. By Corollary 2.4, x is a place and y is a transition. It is similarly shown that $\varphi(x) \in N'_1$ and $\varphi(y) \notin N'_1$ implies that x is a transition and y is a place. The result follows by the definition of a transition-bordered subnet. ■

3 Transformation of S- and T-components

Recall from the introduction that an S-component of a net yields a data-oriented view of a part of the system. An S-component can contain nondeterministic choices that are modeled by branching places, i.e. by places with more than one output transitions. It does however not contain aspects of concurrency, whence its transitions are not branched. Similarly, T-components concentrate on functional aspects. They do not contain branching places. Formally S-components and T-components are particular subnets.

Definition 3.1

A strongly connected transition-bordered subnet N_1 of a net N is called *S-component* of N if, for every $t \in T_1$, $|{}^\bullet t \cap S_1| \leq 1 \;\wedge\; |t^\bullet \cap S_1| \leq 1$ (pre-sets and post-sets are taken with respect to N).

A net N is said to be *covered by S-components* if there exists a family of S-components (N_i), $i \in I$, such that for every $x \in X$ there exists an $i \in I$ such that $x \in X_i$.

Definition 3.2

A strongly connected place-bordered subnet N_1 of a net N is called *T-component* of N if, for every $s \in S_1$, $|{}^\bullet s \cap T_1| \leq 1 \;\wedge\; |s^\bullet \cap T_1| \leq 1$ (pre-sets and post-sets are taken with respect to N).

A net N is said to be *covered by T-components* if there exists a family of T-components (N_i), $i \in I$, such that for every $x \in X$ there exists an $i \in I$ such that $x \in X_i$.

The net shown in Figure 7 is covered by S- and by T-components.

Definition 3.3

Let $\varphi: N \rightarrow N'$ be a net homomorphism and let N_1 be a subnet of N.

The net $(\varphi(X_1) \cap S', \varphi(X_1) \cap T'; \{(\varphi(x), \varphi(y)) \mid (x,y) \in F_1 \wedge \varphi(x) \neq \varphi(y)\})$ is called the *net image of N_1 by φ*. It is denoted by $\varphi(N_1)$.

By $\varphi_{N_1}: X_1 \rightarrow \varphi(X_1)$ we denote the restriction of φ to X_1, with the range of φ restricted to $\varphi(X_1)$; φ_{N_1} is surjective by definition.

Note that $\varphi(N_1)$, the net image of N_1, is not necessarily a subnet of the target net N'. Figure 8 gives an example.

Using Definition 3.3 we get immediately the following results:

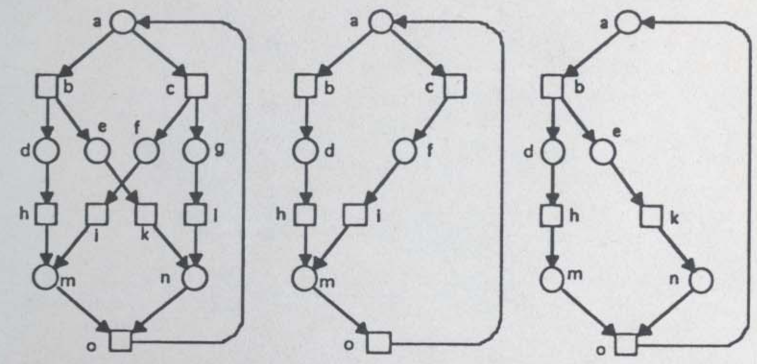

Figure 7: *A net with one of its two S-components and one of its two T-components; there is only one family of S-components and one family of T-components which cover the net*

Proposition 3.4

If $\varphi: N \rightarrow N'$ is a net homomorphism and N_1 is a subnet of N then $\varphi_{N_1}: N_1 \rightarrow \varphi(N_1)$ is a quotient. ■

Corollary 3.5

A net homomorphism $\varphi: N \rightarrow N'$ is a quotient if and only if $N' = \varphi(N)$ and in this case $\varphi = \varphi_N$. ■

S-vicinity respecting net homomorphisms map a strongly connected transition-bordered subnet either onto a single element or onto a strongly connected transition-bordered subnet.

Proposition 3.6

Let $\varphi: N \rightarrow N'$ be an S-vicinity respecting net homomorphism and let N_1 be a strongly connected transition-bordered subnet of N. Define $N_1' = \varphi(N_1)$. Then:

1. *N_1' is a subnet of N';*
2. *If $|X_1'| > 1$ then N_1' is a transition-bordered subnet of N';*
3. *$\varphi_{N_1}: N_1 \rightarrow N_1'$ is S-vicinity respecting.*

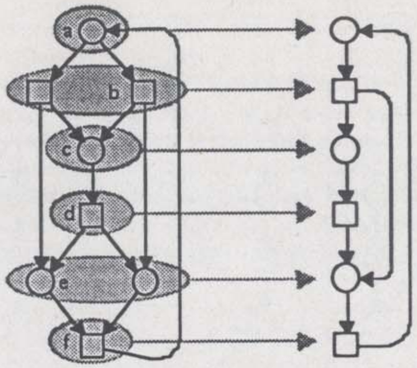

Figure 8: *The image of the subnet generated by $\{a,b,c,d,e,f\}$ is not a subnet of N' because of the arc from the image of b to the image of e*

Proof: Assume $|X_1'| > 1$ (otherwise the proposition trivially holds).
1. Obviously $S_1' \subseteq S'$, $T_1' \subseteq T'$ and $F_1' \subseteq F' \cap ((S_1' \times T_1') \cup (T_1' \times S_1'))$.
Let $x', y' \in X_1'$ such that $(x',y') \in F'$. We show that $(x',y') \in F_1'$.
Assume that x' is a place. $\varphi_{N_1}: N_1 \to N_1'$ is a surjective net homomorphism. By Lemma 1.13 we can find an arc $(x,z) \in F_1$ such that $\varphi_{N_1}(x) = x'$ and $\varphi_{N_1}(z) \neq x'$. By Corollary 2.4, $x \in S_1$. Since φ is S-vicinity respecting and $\varphi(x^{\odot}) \neq \{\varphi(x)\}$ ($\odot$-notation w.r.t. F) there exists some $y \in T_1$ with $y \in x^\bullet$ and $\varphi(y) = y'$ since N_1 is a transition-bordered subnet. Hence $(x,y) \in F_1$ and $(\varphi(x), \varphi(y)) = (x', y') \in F_1'$.
The case $y' \in S'$ is analogous.
2. We only show $S_1'^\bullet \subseteq T_1'$, ${}^\bullet S_1' \subseteq T_1'$ being similar.
Let $x' \in S_1'$, $y' \in T'$ such that $(x',y') \in F'$.
Arguing like above, we can find an $x \in S_1$ with $\varphi_{N_1}(x) = x'$ and some $y \in x^\bullet$ with $\varphi(y) = y'$. We have $y \in T_1$ because ${}^\bullet S_1 \cup S_1^\bullet \subseteq T_1$. Hence $y' \in T_1'$, which was to prove.
3. Let $x \in S_1$. We show $\varphi_{N_1}({}^{\odot}x) = \{\varphi_{N_1}(x)\}$ or $\varphi_{N_1}({}^{\odot}x) = {}^{\odot}(\varphi_{N_1}(x))$.
If $\varphi({}^{\odot}x) = \{\varphi(x)\}$ then $\varphi_{N_1}({}^{\odot}x) = \{\varphi(x)\} = \{\varphi_{N_1}(x)\}$ and we are done.
Otherwise, since ${}^{\odot}x \subseteq X_1$ we have $\varphi_{N_1}({}^{\odot}x) = \varphi({}^{\odot}x)$. Since φ is S-vicinity respecting $\varphi({}^{\odot}x) = {}^{\odot}(\varphi(x))$. Since N_1' is a transition-bordered subnet by 2., ${}^{\odot}\varphi(x) \subseteq X_1'$. Therefore ${}^{\odot}(\varphi(x)) = {}^{\odot}(\varphi_{N_1}(x))$. ∎

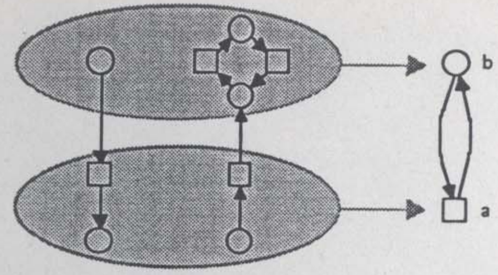

Figure 9: *The image of the left connected subnet by the S-vicinity respecting quotient is not a subnet because of the arc (a,b)*

The example in Figure 9 shows that being strongly connected is a necessary prerequisite for Proposition 3.6. In Figure 8 we gave an example of a strongly connected subnet which is not a transition-bordered subnet. Its image by the S-vicinity respecting quotient is not a subnet of the target net.

An S-component is a special strongly connected transition-bordered subnet. For respecting coverings by S-components, stronger hypotheses have to be assumed. Let us continue considering the S-vicinity respecting quotient shown in Figure 10.

The net N on the left hand side is covered by S-components. The net homomorphism φ is an S-vicinity respecting quotient. However, the target net is not covered by S-components. Observe that the restriction of φ to any S-component is not T-vicinity respecting. Consider the S-component N_1 containing b. The image of N_1 is the entire target net. We have $\varphi_{N_1}(\{a,b\}) \neq \{\varphi_{N_1}(a)\} = \{u\}$ but $\varphi_{N_1}(\{a,b\}) = \{u,w\} \neq (\varphi_{N_1}(a))^{\odot} = \{u,v,w\}$. The net image of N_1 is not an S-component of the target net.

In Figure 11, the quotients restricted to any S-component are T-vicinity respecting. Remember that quotients can be simply drawn by depicting the equivalence classes of elements that are identified by the quotient, as shown for arbitrary graphs in section 1.

Proposition 3.7

Let $\varphi: N \to N'$ be an S-vicinity respecting net homomorphism and let N_1 be an S-component of N. Define $N_1' = \varphi(N_1)$ and suppose that $\varphi_{N_1}: N_1 \to N_1'$ is T-vicinity respecting. If $|X_1'| > 1$ then N_1' is an S-component of N'.

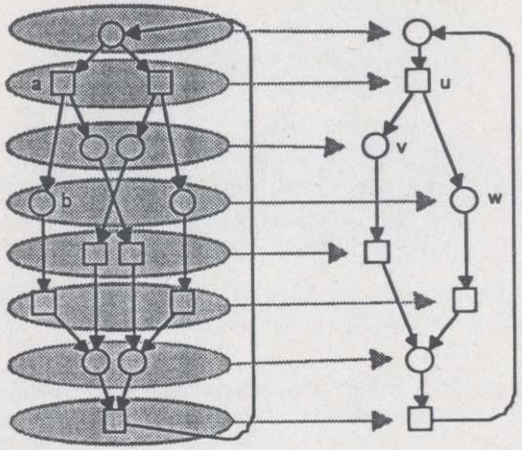

Figure 10: *An S-vicinity respecting quotient which is not T-vicinity respecting if restricted to an S-component*

Proof: Assume $|X_1'| > 1$. Since N_1 is an S-component, it is a transition-bordered subnet. Hence, by Proposition 3.6, N_1' is a transition-bordered subnet of N'.

It remains to prove: every $y' \in T_1'$ satisfies $|{}^\bullet y' \cap S_1'| \le 1$ and $|y'^\bullet \cap S_1'| \le 1$. Let $y' \in T_1'$. We show only: $|{}^\bullet y' \cap S_1'| \le 1$ ($|y'^\bullet \cap S_1'| \le 1$ is similar). Since φ_{N_1} is surjective we can find an arc $(x,y) \in F_1$ with $\varphi(x) \neq y'$ and $\varphi(y) = y'$ (Lemma 1.13). Since φ_{N_1} is a T-vicinity respecting quotient, Corollary 1.15 implies $|S_1' \cap {}^\bullet y'| \le |S_1 \cap {}^\bullet y|$ and $|S_1 \cap {}^\bullet y| \le 1$ because N_1 is an S-component. ∎

From Proposition 3.7 we deduce:

Theorem 3.8

Let N be a net which is covered by a family $(N_i), i \in I$ of S-components. Let $\varphi: N \to N'$ be an S-vicinity respecting quotient such that, for all $i \in I$, $\varphi_{N_i}: N_i \to \varphi(N_i)$ is T-vicinity respecting. Then N' is covered by S-components.

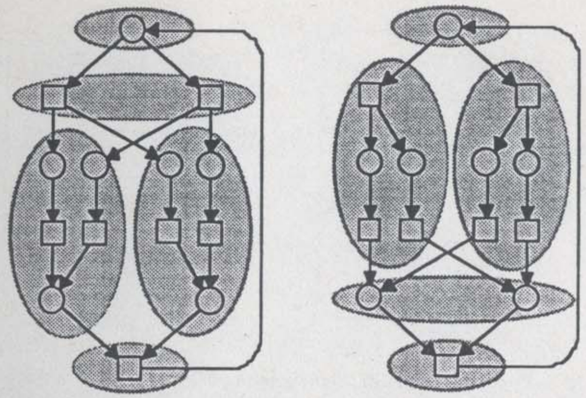

Figure 11: *S-vicinity respecting quotients*

Proof: In Proposition 3.7 we have shown that, given the assumptions above, every S-component of N is either mapped to an S-component of N' or to an element of N'.

Let $x' \in X'$. If x' is an isolated element then it is a trivial S-component. So assume that x' is not isolated. Then we can find a $y' \in X'$ such that $(x',y') \in F'$ or $(y',x') \in F'$.

Since φ is a quotient, there are $x \in S, y \in T$ with $\varphi(\{x,y\}) = \{x',y'\}$ and either $(x,y) \in F$ or $(y,x) \in F$. N is covered by S-components and hence we can find an $i \in I$ such that $x \in S_i$ and $y \in T_i$. $|X_i'| > 1$ since x' and y' are distinct elements of $\varphi(N_i)$. Thus $\varphi(N_i)$ is an S-component of N'. ∎

By Proposition 3.6 (3) 'φ is S-vicinity respecting' implies for all $i \in I$: 'φ_{N_i} is S-vicinity respecting'. So all the φ_{N_i} have to be both S- and T-vicinity respecting. However, this alone does not imply that φ is S-vicinity respecting and is not sufficient for N' to be covered by S-components as is shown in Figure 12. For the S-components N_1 of the net N of Figure 12 φ_{N_1} is S- and T-vicinity respecting. However, φ is not S-vicinity respecting and N' is not covered by S-components.

Theorem 3.8 implies that, given a family of S-components which cover the source net, a respective covering of the target net is obtained by the images of the S-components which are not mapped to single non-isolated places.

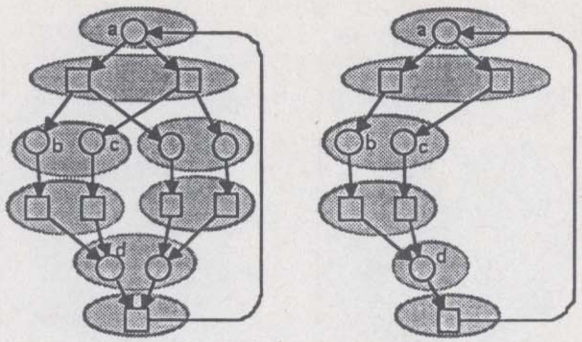

Figure 12: *A net N and an S-component N_1 of N. The target net of the depicted net homomorphism is neither covered by T-components nor by S-components*

The choice of a covering family of S-components is decisive. In the example of Figure 13, the quotient is vicinity respecting. Its restriction to either the S-component N_1 which contains a_1, a_2 or to the S-component N_2 which contains a'_1, a'_2 is T-vicinity respecting. Taking the S-components N_3 and N_4 as a cover of N, with a_1, a'_2 belonging to N_3 and a'_1, a_2 belonging to N_4, the restriction of φ to any of these S-components is not T-vicinity respecting. This example points out that the choice of an abstraction and the choice of an S-component covering are not independent.

By duality we get:

Corollary 3.9

Let $\varphi: N \to N'$ be a T-vicinity respecting net homomorphism and let N_1 be a strongly connected place-bordered subnet of N. Define $N'_1 = \varphi(N_1)$. Then:

1. *N'_1 is a subnet of N';*
2. *If $|X'_1| > 1$ then N'_1 is a place-bordered subnet of N';*
3. *$\varphi_{N_1}: N_1 \to N'_1$ is T-vicinity respecting.*

The dual version of Theorem 3.8 reads as follows:

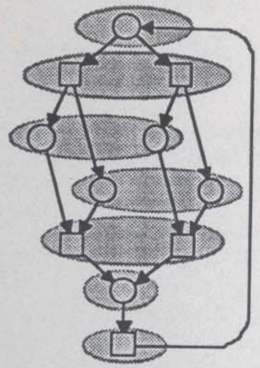

Figure 13: *This net has two coverings by S-components. The S-vicinity respecting quotient is such that its restriction to the S-components of one covering is T-vicinity respecting while this does not hold for the other covering*

Theorem 3.10

Let N be a net which is covered by a family $(N_i), i \in I$ of T-components. Let $\varphi: N \to N'$ be a T-vicinity respecting quotient such that, for all $i \in I$, $\varphi_{N_i}: N_i \to \varphi(N_i)$ is S-vicinity respecting. Then N' is covered by T-components.

The net homomorphisms depicted in Figure 11 are vicinity respecting. Their restrictions to any S-component or T-component are also vicinity respecting. Hence their net images are covered by S- and T-components.

A particular case of Theorem 3.8 is the composition of S-components; the source net N is the disjoint union of a family of S-components and the mapping, restricted to each of these S-components, is injective (and hence a fortiori T-vicinity respecting).

We can reformulate our result as a property of net homomorphisms as follows:

For every place a of an S-component N_1 of a net N the entire vicinity belongs to the S-component as well by definition. Therefore the natural injection $\psi_1: N_1 \to N$ is S-vicinity respecting but not necessarily surjective.

A covering by S-components N_i $(i \in I)$ can be expressed by a set of net homomorphisms ψ_i $(i \in I)$ such that each element of N is in $\psi_i(N_i)$ for at least one i. Using the disjoint union of the S-components $(\biguplus N_i)$, the net homomorphisms ψ_i induce a quotient ψ from $\biguplus N_i$ to N.

Now Theorem 3.8 reads as follows. Given

- a family $(N_i), i \in I$ of strongly connected nets with $|{}^\bullet t| \leq 1$, $|t^\bullet| \leq 1$ for all transitions t (S-components),
- S-vicinity respecting injective net homomorphisms $\psi_i: N_i \to N (i \in I)$ such that the induced mapping ψ is a quotient (i.e., N is covered by the N_i),
- an S-vicinity respecting quotient $\varphi: N \to N'$ such that φ_{N_i} is T-vicinity respecting for all $i \in I$,

we can find injective S-vicinity respecting mappings $\psi_i': \varphi(N_i) \to N'$ such that the induced mapping $\psi': \bigcup \varphi(N_i) \to N'$ is surjective (i.e., N' is covered by the $\varphi(N_i)$).

An example is shown in Figure 14. Again, by duality we can use the same formalism to capture the composition of T-components.

4 Siphons, Traps and Free-Choice Property

We turn now to the structural concepts called siphons and traps. As the names suggest, a trap is a set of places where any output transition is also an input transition of one of the places while any input transition is also an output transition for the places of a siphon.

Definition 4.1

A siphon of a net is a nonempty set of places A satisfying ${}^\bullet A \subseteq A^\bullet$.

A trap of a net is a nonempty set of places A satisfying $A^\bullet \subseteq {}^\bullet A$.

For marked Petri nets, siphons and traps are used to deduce behavioural properties of the system. Also at the conceptual level of Channel/Agency nets, they can be used to analyze aspects of the data and information flow in the modeled system. Roughly speaking, if a set of places is a trap then information cannot get completely lost in the component modeled by these places. For the places of a siphon, it is not possible to add information without taking data from the siphon into account.

As an example, consider the net shown in Figure 15. The set of places $\{a, d, e\}$ is a trap. Since any output transition of a trap is also an input transition, no transition can use data from places of this set without creating new data for a

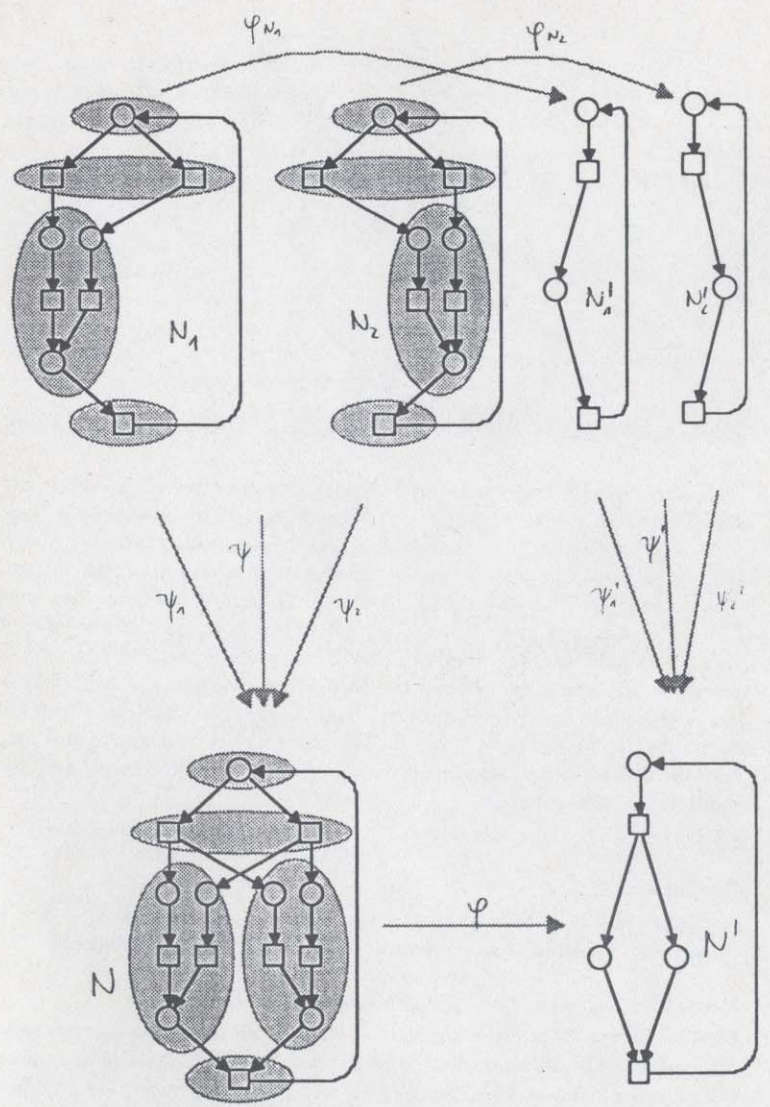

Figure 14: *An example of the commutativity of composition and coarsening of nets*

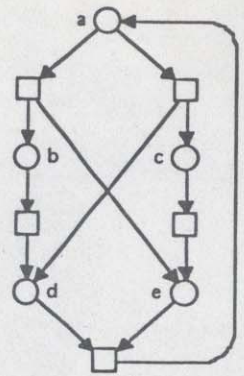

Figure 15: *The set $\{a, d, e\}$ is a trap; the set $\{a, b, d\}$ is a siphon and a trap*

place of this set. A trap can be used to ensure that a trace of some data will always remain in a system component. The set $\{a, b, d\}$ is a siphon and a trap. Since it is a siphon, any input transition is also an output transition. Therefore, no place of this set can receive data without taking data into account that is already contained in the set. A siphon can be used to ensure that some data will not enter a system component. A siphon (trap) is minimal when it does not strictly include any other siphon. The traps $\{a, d, e\}$ and $\{a, b, d\}$ of the previous example are minimal and $\{a, b, d\}$ is also a minimal siphon. Minimal siphons and traps are particularly important for the analysis of marked Petri nets [7]. We will show that vicinity respecting net homomorphisms map minimal siphons either onto singletons or onto siphons of the target net, and similarly for minimal traps.

We begin with a preliminary result (the proof follows the one given in [7]):

Proposition 4.2

Let A be a minimal siphon of a net N, i.e. no proper subset of A is a siphon. Then the subnet generated by $^\bullet A \cup A$ is strongly connected.

Proof: Let $N_A = (A, {}^\bullet A; F_A)$ be the subnet generated by A.

First we observe that every transition t of N_A is an input transition of some place of A by the definition of N_A and also an output transition of some place of A because A is a siphon. Hence, for proving strong connectivity it suffices to show that for every two places $x, y \in A$ there is a path $x \dots y$ in N_A.

Let $y \in A$ and define the set $X = \{z \in A \mid \text{ there is a path } z \dots y \text{ in } N_A\}$. We prove that $X = A$. This implies $x \in X$ and, by the definition of X, proves the result we are after.

Let $t \in {}^\bullet X$. Since $X \subseteq A$ and since A is a siphon we have $t \in A^\bullet$. By the definition of X there is a path $t \dots y$ in N_A. So every place in ${}^\bullet t \cap A$ belongs to X. Therefore $t \in X^\bullet$.

So we have ${}^\bullet X \subseteq X^\bullet$. X is not the empty set because $y \in X$. So X is a siphon included in A. Since A was assumed to be a minimal siphon we conclude that $X = A$. ■

Theorem 4.3

Let $\varphi: N \to N'$ be a surjective S-vicinity respecting net homomorphism. If A is a minimal siphon of N then either $\varphi(A)$ is a single node (place or transition) or $\varphi(A) \cap S'$ is a siphon of N'.

Proof: Since A is a minimal siphon, $^\bullet A \cup A$ generates a strongly connected subnet N_1 by Proposition 4.2. Hence the subnet N_1' of N' generated by the set of elements $\varphi({}^\bullet A) \cup \varphi(A)$ is also strongly connected.

Assume that $\varphi(A)$ contains more than one node. Let x' be a place of N_1' and let $z' \in {}^\bullet x'$. We have to prove that z' has an input place of N_1'.

Since N_1' is strongly connected and since it contains more than one element, it contains a transition $y' \in {}^\bullet x'$. So there exist a place $x \in A$ and a transition $y \in {}^\bullet x$ such that $\varphi(x) = x'$ and $\varphi(y) = y'$. In particular $\varphi({}^\circleddash x) \neq \{\varphi(x)\}$. Since φ is S-vicinity respecting, we obtain $\varphi({}^\circleddash x) = {}^\circleddash \varphi(x)$. This implies that some $z \in {}^\bullet A$ is mapped to z'. Since A is a siphon, z has an input place of A. So z' has an input place of N_1' which completes the proof. ■

By symmetrical arguments, the results proved for siphons also hold for traps:

Proposition 4.4

Let A be a minimal trap of a net N. Then the subnet generated by $A \cup A^\bullet$ is strongly connected. ■

Theorem 4.5

Let $\varphi: N \to N'$ be a surjective S-vicinity respecting net homomorphism. If A is a minimal trap of N then either $\varphi(A)$ is a single node (place or transition) or $\varphi(A) \cap S'$ is a trap of N'. ■

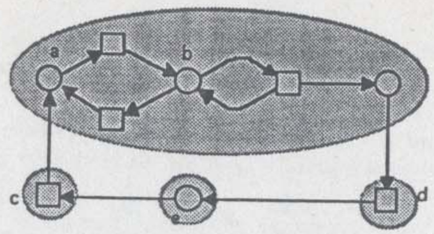

Figure 16: *The trap $\{a,b\}$ is mapped onto a single element; its image is not a trap of the target net*

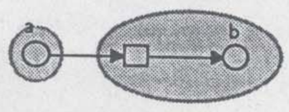

Figure 17: *The trap $\{a,b\}$ is not strongly connected; its image is not a trap of the target net*

Figure 16 and Figure 17 show that the strong connectedness of ${}^{\bullet}A \cup A$, implied by its minimality, and the fact that the image of A does have more than one element are necessary conditions.

We close this section establishing that vicinity respecting quotients respect free choice Petri nets. Important behavioural properties are characterized in terms of traps, siphons for these nets and the class of free choice Petri nets which is covered by S- and T-components is well established [7]. In a free choice net, if two transitions share some input places, then they share all their input places.

Definition 4.6

A net N is called free choice if for any two places s_1 and s_2 either $s_1^{\bullet} \cap s_2^{\bullet} = \emptyset$ or $s_1^{\bullet} = s_2^{\bullet}$.

The net shown in Figure 7 is free choice.

Theorem 4.7

Let N be a free choice net and φ: $N \rightarrow N'$ be a vicinity respecting quotient. Then N' is free choice as well.

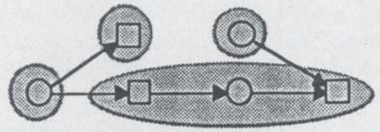

Figure 18: *This quotient is S-vicinity respecting only, the target net is not free choice.*

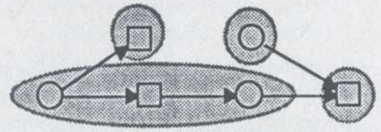

Figure 19: *This quotient is T-vicinity respecting only, the target net is not free choice.*

Proof: We show that for any two places s'_1, s'_2 of S' we have: $s'^{\bullet}_1 \cap s'^{\bullet}_2 = \emptyset$ or $s'^{\bullet}_1 = s'^{\bullet}_2$.

We proceed indirectly. Let s'_1 and s'_2 be places of S' such that $s'_1{}^{\bullet} \cap s'_2{}^{\bullet} \neq \emptyset$ and $s'_1{}^{\bullet} \neq s'_2{}^{\bullet}$. Without loss of generality assume that there is a transition $t'_2 \in s'^{\bullet}_2$ with $t'_2 \notin s'^{\bullet}_1$. Let $t'_1 \in s'_1{}^{\bullet} \cap s'_2{}^{\bullet}$. Since φ is a quotient we find $(s_2, t_1) \in F$ with $\varphi(s_2) = s'_2$ and $\varphi(t_1) = t'_1$. Since φ is S-vicinity respecting we have $\varphi(s_2{}^{\odot}) = \varphi(s_2)^{\odot}$. Hence there is a transition $t_2 \in s_2{}^{\bullet}$ satisfying $\varphi(t_2) = t'_2$. Since φ is T-vicinity respecting, $\varphi({}^{\odot}t_1) = {}^{\odot}\varphi(t_1)$. Hence there is a place $s_1 \in {}^{\bullet}t_1$ with $\varphi(s_1) = s'_1$. Since φ is vicinity respecting and since $t'_2 \notin s'_1{}^{\bullet}$ we get $t_2 \notin \varphi(s_1{}^{\bullet})$ – a contradiction to the free-choice property of N. ∎

Figure 18 and 19 show that for the previous theorem S-vicinity respecting and T-vicinity respecting alone are not sufficient.

5 Conclusion

Structuring software requirements is a gradual process which involves refinement/abstraction between different conceptual levels. Abstractions should bear formal relations with refinements because otherwise the analysis of some abstraction will be of no help for the induced refinement. In section 2 we argued that vicinity respecting homomorphisms give a possible and – to some extend – satisfactory solution to these requirements for graph-based models of distributed systems. They provide a method to perform graphical abstraction/refinement such that every element is either glued together with its vicinity or its vicinity is the vicinity of its image.

The vicinity respecting concept is a local notion because its definition only uses local vicinities. However, it has global consequences since it preserves paths and, consequently, connectedness properties. For Petri nets, vicinity respecting homomorphisms preserve moreover important structural properties such as S- and T-invariants, siphons and traps and the free-choice property – concepts that are also defined employing local vicinities of single elements.

Our starting point was the graph homomorphism notion [19], which is slightly relaxed, and the net morphism notion as presented in [20,12]. We are concerned with the gradual structuring of software requirements. After sufficient steps of refinement and completion a Channel/Agency Petri Net is constructed that contains all relevant details. At this level, a marking can be added to the net. It fixes the resources which are factually available. Now knowledge about S- and T-components, traps and siphons can be used while simulating and analyzing the marked net with computer tools [21].

Other concepts for refinement and abstraction of Petri nets (see [3] for an overview) and of morphisms [24,18] have been proposed in the literature. However, all these approaches are concerned with marked Petri nets and aim at results involving the behaviour given by the token game. In contrast, we are concerned with the preliminary task of structuring software requirements down to a working system and aim at structure preservation. Generally, abstraction in our sense is more general than behaviour preserving abstraction. However, structure influences behaviour. The transition refinement considered in [13] turns out to induce a vicinity respecting homomorphism from the refined net to the coarser net.

Structuring software requirements is an emerging issue in software engineering [15]. Many methodologies use at some point graphical representations of their systems. However, most of them rely on a language approach or an algebraic approach to formally describe the system and to define abstraction/refinement mechanisms, see [23], [16], [4] for examples. An originality of the Petri Net approach is to use graphs not only as a friendly visual support but also as a formal mathematical model.

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