# Fuzzy Diagnosis

(Editorial)

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## 1 Introduction

Starting from the pioneering publication of Lotfi Zadeh in 1965 [55], fuzzy sets have been applied to many fields in which uncertainty plays a key role. Medicine, often on the borderline between science and art, is an excellent exponent: vagueness, linguistic uncertainty, hesitation, measurement imprecision, natural diversity, subjectivity – all these are prominently present in medical diagnosis.

While statistical uncertainty can be handled in a rigorous way, the treatment of *nonstatistical* uncertainty is still a challenge [22]. For example, the nonstatistical uncertainty in "high" attributed to the blood pressure of a patient is at least threefold:

- **The patient.** For a normally hypotonic patient "high" blood pressure is something quite different from what it is for a normally hypertonic patient.
- **The expert.** Different experts have different opinions about what values of the variable *blood pressure* should be called "high".
- **The diagnostic problem.** Definitions depend on the medical context. During anaesthesia for example, what is labeled "high" blood pressure may well be "normal" in another context.

In all three cases there is no uncertainty associated with the value as it is, e.g., 145 mmHg. Why, then, do we need to enforce imprecise linguistic attributes such as "high"? Because in certain situations, of the two sentences

- The systolic arterial pressure of $X$ is 145 mmHg.
- The systolic arterial pressure of $X$ is *high*.

the second is more adequate than the first.

Fuzzy set theory is a response to the demand for ideas and approaches for handling nonstatistical uncertainty. The initial enthusiasm about fuzzy sets was associated with its ability to model linguistic terms and expressions. It promised to bring automated reasoning and human thinking, which until then had almost exclusively been linked by predicate calculus, closer together. However, linguistic modeling and approximate reasoning are not the only ways to use fuzzy sets. The degree of membership can be used to express, for example, the degree of occlusion of a coronary vessel, the involvement of a lymph node, the degree of abnormality of a certain ECG episode, etc. This second use of fuzzy sets seems to be gaining momentum at present.

Indeed, there appears to be a certain shift in what fuzzy sets are most approved for. Initially, fuzzy sets were integrated into rule-based expert systems with the intent to remedy the "brittleness" of traditional AI decision support [30, 7]. Typically, the source of data was the patient record [12, 6, 52], and this data was "fuzzified" and processed by a fuzzy inference machine, e.g., by relational calculus [1, 43, 41, 44, 51, 50] or by some other heuristic scheme [36, 42].

The hallmarks of this first phase are:

- The main source of data was the patient record. The computational technology was not efficient enough for fuzzy signal or image processing.
- Most of the fuzzy aid in medicine was linked with AI, and fuzzy sets were used to model linguistic terms and verbal expressions.
- There was no automatic derivation or tuning of membership functions. Usually these functions were provided by the domain expert or set up heuristically (using common sense) by the system designer.

The second phase is characterized by:

- Automatic generation and tuning of membership functions. The domain expert is less involved in the system design. There is a strong trend towards extracting the rules and the membership functions of the knowledge base from data.

- Signal and image processing using fuzzy sets have come to the fore [4, 17, 5, 23, 40, 27, 24].

- Merging AI and pattern recognition for medical decision support has been recognized (at a philosophical level) as a promising research direction [33, 14]. Now, fuzzy pattern recognition and fuzzy control come more into play, making extensive use of fuzzy clustering and fuzzy neural networks.

There is a variety of medical fields where fuzzy sets have been applied. Recently, a great deal of work has been done in anaesthesia monitoring [54, 45, 28, 3, 31, 2] and cardiology [16, 46, 35, 25, 21, 18, 19, 15, 9, 48, 39]. There are many more niches of medical science where fuzzy sets appear to be an appealing option [20, 53, 8, 49, 26, 37, 47, 11, 13], and this variety is rapidly growing. *Fuzzy diagnosis*, however, is the original application domain of fuzzy set theory in medicine, and this special issue is entirely dedicated to this subject.

## 2 Fuzzy Diagnosis

The problem of medical diagnosis can be formalised as follows. Let $\mathbf{C} = \{C_1, C_2, \ldots, C_M\}$ be a set of $M$ diagnoses possible in the context of a certain medical problem. $\mathbf{C}$ can be: a list of disorders, types of tissues in a Magnetic Resonance (MR) scan of the brain, types of blood cells, etc. We call $\mathbf{C}$ a set of class labels. Let $\mathbf{x}$ be the description of an object (e.g., a patient, a piece of brain tissue, a cell) in the form of an $n$-dimensional real vector $\mathbf{x} = [x_1, \ldots, x_n]^T \in \Re^n$. The components of $\mathbf{x}$ encode the features, e.g., clinical measurements and findings; details from patient's history; physiological parameters; test results; image parameters like grey level intensity, "roundness" of the cell, etc. A classical *classifier* is any mapping

$$D : \Re^n \to \mathbf{C}.$$

That is, for every object $\mathbf{x} \in \Re^n$, the classifier specifies a single class label which is interpreted as the diagnosis.

*Fuzzy diagnosis* is characterized by the fact that the classifier relies on fuzzy sets for solving a medical diagnostic problem. Fuzzy sets can be used at different stages of the classifier design, and in different ways. The most apparent are

- **Fuzzy inputs.** Instead of the original input values (e.g., measurements) their "fuzzified" versions can be used. For example, instead of a value 145 mmHg for the blood pressure, we can use the vector $[0.0, 0.4, 0.6]^T$ consisting of the degrees of membership of that value to the fuzzy sets $[low, medium, high]$.

- **Fuzzy reasoning.** The implementation of the classifier is based on fuzzy sets, for example on a fuzzy inference machine.

- **Fuzzy classes.** Classical pattern recognition assumes that the classes are mutually exclusive. This, however, is not generally the case in medical diagnosis. Instead, some of the disorders can occur simultaneously in the same patient, but with varying degrees. Therefore, each patient can be labeled in more than one class.

While fuzzy input and fuzzy reasoning are technicalities that may or may not interest the user, fuzzy classes have a marked effect on medical diagnosis. Instead of $D(\mathbf{x}) \in \mathbf{C}$, a *fuzzy classifier* performs the mapping

$$\tilde{D} : \Re^n \to [0,1]^M,$$

i.e., $\tilde{D}(\mathbf{x}) = [\mu_1(\mathbf{x}), \ldots, \mu_M(\mathbf{x})]^T$, where $\mu_i(\mathbf{x})$ denotes the degree to which $\mathbf{x}$ belongs in class $C_i$. This degree can be interpreted in many different ways, the most conventional of which are:

- **Typicality** of case $\mathbf{x}$ with respect to diagnosis $C_i$.

- **Severity** of disorder $C_i$ in $\mathbf{x}$.

- **Support** for the hypothesis that $C_i$ is the true diagnosis for $\mathbf{x}$, deduced from the available evidence.

- **Probability** that $C_i$ is the true diagnosis for $\mathbf{x}$.

An enlightening discussion on three basic semantic categories of degrees of membership: *similarity, preference*, and *uncertainty* is presented in [10].

The fuzzy decision $\tilde{D}(\mathbf{x})$ can be "hardened" to get one single class label from $\mathbf{C}$. Usually the most "supported" class is chosen (called *maximum membership rule*), i.e.,

$$D(\mathbf{x}) = C_j \in \mathbf{C} \iff \mu_j(\mathbf{x}) = \max_i \mu_i(\mathbf{x}).$$

# 3 Fuzzy versus crisp diagnosis

Paradoxically, most diagnostic fuzzy systems indeed "defuzzify" their output. While this may be a technical necessity in fuzzy control, it is certainly contrary to the original conception that fuzzy reasoning, modelled after human thinking, is more adequate in uncertain domains like medicine. If we really need crisp answers, then why bother using fuzzy sets in the first place?

Do we need crisp answers? Sometimes, maybe, yes, if the choices for action that we have are also crisp. But do we need crisp answers from a diagnostic aid? We contend no! The indecision generally associated with medical decision making lies in the nature of the problem, and no however sophisticated algorithm can compute it away. Giving crisp answers on uncertain grounds is a dangerous delusion that cannot be justified by whatever practical desires.

Why, then, do we prefer crisp answers over fuzzy ones? The reasons are manyfold. First and foremost, our striving for rigour has so much pervaded our scientific thinking that we find it unacceptable to not know what to do with a case. How can I justify my not knowing? What do I tell the patient or my peer? How do I evaluate my indecision against a gold standard which is always decided (albeit, sometimes, wrong)? While these are certainly pressing questions for the human decision maker, it should be clear that we cannot delegate the final responsibility of overruling the pros and cons for one or the other option to a machine.

Secondly, the choice of a therapy may force us assume a certain diagnosis. A patient either undergoes surgery, or not. In many other cases, however, a graded diagnosis allows the treatment to be adapted to the individual's needs. For example, it may give us the chance to adjust the dose of a medication to the degree of illness: to treat mild hypertension, a low dosage may suffice. This, the principle of fuzzy control, will be the key to the success of many future fuzzy medical systems.

Finally, clinical evaluation, the prerequisite for acceptance and widespread dissemination of any diagnostic aid, of a fuzzy system is yet unclear. How do we set up a fuzzy confusion matrix? How are sensitivity, specificity, and accuracy of a fuzzy diagnosis defined? Is the gold standard itself naturally crisp, or has it inadequately been forced to be so? And with the fuzzy measures given, will fuzzy systems demonstrably do better than their crisp pendants?

Answers to these last two questions are particularly hard to give. But only they will eventually disclose the true value of fuzziness in medical diagnosis.

## 4 Transparent versus accurate diagnosis

Another important choice in the design of diagnostic systems is that between transparency and accuracy. Transparency (or linguistic interpretability) is highly desirable in medical decision support. Typically, a diagnostic system should be able to "explain" how it operates in the language of the domain professionals. This requires that the knowledge base is kept simple and comprehensible. The more complex a system gets, however, the more transparency contradicts accuracy, until a point is reached at which they are almost mutually exclusive properties. The classical utilization of fuzzy sets is in capturing complex relationships with transparent representations. The concomitant loss of accuracy is considered a small price given the enormous problems encountered in establishing accurate specifications.

Once we leave the scope of linguistic representations and approximate reasoning, however, we find that we can *gain* diagnostic accuracy using fuzzy sets. The concept of *fuzzy diagnosis* adopted here encompasses more than using fuzzy sets as linguistic terms or degrees of certainty. Fuzzy sets can be used within black boxes – like any other abstract mathematical means – for *calculating* rather than *inferencing* the diagnostic label. This facet of fuzzy diagnosis is representative of the second phase pointed out above, and aims at higher diagnostic accuracy, disregarding the interpretability issue.

The careful design of a diagnostic system involves many trade-offs. The magic word "fuzzy" alone does not guarantee an improvement over conventional systems. Rather, we have to make sure that our model gains in either transparency, accuracy, simplicity, or another quality, as compared to various other readily available, easy-to-design and highly accurate non-fuzzy models.

## 5 About this issue

The four papers in this special issue represent the second phase outlined in the introduction:

- Tuning membership functions by neural networks [34, 32].
- Diagnosing medical signals [29] and images [32, 38].
- Moving from linguistic modeling [29, 34] towards nonlinguistic applications of fuzzy sets [38, 32].

The paper by Masulli and Schenone [32] describes a method for segmentation of Magnetic Resonance Images (MRI). The classes $C_i$ are various types of tissues in images of human brain: white and grey matter, cerebro-spinal fluid, eyes, and other structures of interest. The classifier $\tilde{D}$ is based on fuzzy clustering. Each voxel $\mathbf{x}$ in the image (a point in $\Re^3$) is labeled in all classes, and the final class label is found by the maximum membership rule. The authors design $\tilde{D}$ using the Capture Effect Neural Network (CENN) with a version of possibilistic fuzzy $c$-means, and use a novel heuristic to eliminate redundant clusters. On two MRI examples they show that their combined technique, called Possibilistic Neuro-Fuzzy C-Means (PNFCM), better matches human opinion about labeling the tissues than either FCM or CENN alone.

Nauck and Kruse [34] use a fuzzy neural network NEFCLASS to extract a knowledge base from data and tune the membership functions automatically, using only the available data set. The authors explain the rationale for the adopted approach and the operation of NEFCLASS. A typical problem in this type of automatic rule-extraction systems is the large number of rules generated. The authors propose an original technique for pruning the rule-base and simultaneously selecting features. The result is a compact fuzzy rule-based classifier with acceptable classification accuracy as an example of balancing transparency and accuracy. By design the membership functions for the linguistic terms used by $\tilde{D}$, e.g., "low", "high", "normal" comply with a set of constraints to assure integrity, plausibility and interpretability of the obtained rule-base. The proposed model is illustrated with a publically available data set.

In his paper [38] Pizzi suggests a technique to fuzzify the original class labels of the data, thereby "burnishing the tarnished gold standard". Let $\mathbf{Z} = \{\mathbf{z}_1, \ldots, \mathbf{z}_N\}, \mathbf{z}_i \in \Re^n$ be the labeled data set that we use to design $\tilde{D}$. Originally, each $\mathbf{z}_i$ is labeled in only one class (i.e., one diagnosis for each object). Pizzi uses a geometrically based technique to assign a fuzzy label to each $\mathbf{z}_i \in \mathbf{Z}$. These labels are then used as the target outputs for training a feed-forward neural network. The idea is illustrated on a set of MR spectra, classified into 3 classes: meninginomas, astrocytomas, and non-tumorous patterns (epilepsy). The fuzzy adjustment of the class labels proposed in the paper compares favourably to other types of fuzzification of the data labels.

Lowe, Harrison and Jones [29] focus on detection of specific problems during monitoring of anaesthesia, e.g., inadequate analgesia, malignant hyperpyrexia, increased intracranial pressure, pulmonary shunt, cardiac output failure, absolute hypovolaemia, and relative hypovolaemia. The problem is to detect the occurrence of any of these conditions during monitoring, and to give a warning or an alarm. The patterns of these classes develop with time, and therefore classical (static) rule-based systems are inadequate. The authors propose to use fuzzy templates: a series of membership values are estimated subsequently in a bout of time during anaesthesia, and are matched to the fuzzy templates. The rationale of this model leads to a fuzzy rule-based system with a fuzzy output $\tilde{D}(\mathbf{x})$. The membership degrees determine the action of the system (alerts and alarms), and the thresholds can be tuned by the user. To verify experimentally their model, the authors use anaesthetic records for about 70 operating hours.

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