Walkshop on Mathematical Physics 2024
Schedule and Abstracts

Organizers: Joachim Kerner and Matthias Täufer

June 13–14, 2024, FernUniversität in Hagen
(Version as of June 12, 2024)

Schedule

Thursday, June 13 2024

9:00 – 9:40	Christian Rose (Potsdam)
9:40 – 10:20	Miłosz Baraniewicz (Wroclav)
10:20 – 10:50	*Coffee*
10:50 – 11:30	Yannick Thomas (Potsdam)
11:30 – 12:10	Liza Schonlau (Bonn)
12:10 – 13:20	*Lunch*
13:20 – 14:00	Patrizio Bifulco (Hagen)
14:00 – 14:40	Kinga Sztonyk (Wroclav)
14:40 – 15:20	Marco Rehmeier (Bielefeld)
15:20 – 15:50	*Coffee*
15:50 – 16:30	Denis Périce (Constructor University Bremen)
16:30 – 17:10	James Kennedy (Lisbon)

Friday, June 14 2024

9:00 – 9:40	Kiyan Naderi (Oldenburg)
9:40 – 10:20	Dennis Schmeckpeper (Hamburg-Harburg)
10:20 – 10:50	*Coffee*
10:50 – 11:30	Badreddine Benhellal (Oldenburg)
11:30 – 12:10	Florian Fischer (Bonn)
12:10 – 13:00	*Lunch*
14:00 – 13:40	Shahnaz Farhat (Constructor University Bremen)
13:40 – 14:20	Illia Karabash (Bonn)
15:00 – 18:30	*Hike*
18:30	Dinner

All talks take place in Room F 009 in Building 3 on the Campus of FernUniversität in Hagen.

On Friday, June 14, we depart from campus via bus 515 at 14:48 and start our hike on the *3-Türme-Weg* ("Three towers trail"). We will arrive at at Hotel & Restaurant Waldlust (Hagen) at 18:30 for dinner.

Abstracts

1 Baraniewicz, Miłosz

Wrocław University of Science and Technology

Estimates of kernels and ground states for Schrödinger Semigroups

We consider the Schrödinger operator of the form $H = -\Delta + V$ acting in $L^2(\mathbb{R}^d, dx)$, $d \geq 1$, where the potential $V : \mathbb{R}^d \to [0, \infty)$ is a locally bounded function. The corresponding Schrödinger semigroup $\left\{e^{-tH} : t \geq 0\right\}$ consists of integral operators, i.e.

$$e^{-tH} f(x) = \int_{\mathbb{R}^d} u_t(x, y) f(y) dy, \quad f \in L^2(\mathbb{R}^d, dx), \ t > 0. \tag{1}$$

I will present new estimates for heat kernel of $u_t(x, y)$. Our results show the contribution of the potential is described separately for each spatial variable, and the interplay between the spatial variables is seen only through the Gaussian kernel.

This estimates will be presented on two common classes of potentials. For confining potentials we get two sided estimates and for decaying potentials we get new upper estimate.

Methods we used to estimated kernel of semigroup allow to easily obtain sharp estimates of ground state for slowly varying potentials.

The talk is based on joint work with Kamil Kaleta [1] and my work [2].

References

[1] M. Baraniewicz, K. Kaleta, *Integral kernels of Schrödinger semigroups with nonnegative locally bounded potentials.* To appear in Studia Mathematica (ArXiv:2302.13886), 2023+.

[2] M. Baraniewicz, *Estmates of ground state for classical Schrödinger operator.* To appear, 2024+.

2 Benhellal, Badreddine

University of Oldenburg

On Neumann-Poincaré operators and self-adjoint transmission problems

In this talk, we discuss the self-adjointness in L^2-setting of the operators acting as $-\mathrm{div} \cdot h\nabla$, with piecewise constant functions h having a jump along a Lipschitz hypersurface Σ, without explicit assumptions on the sign of h. We establish a number of sufficient conditions for the selfadjointness of the operator with H^s-regularity for suitable $s \in [1, \frac{3}{2}]$, in terms of the jump value and the regularity and geometric properties of Σ. An important intermediate step is a link with Fredholm properties of the NeumannPoincaré operator on Σ, which is new for the Lipschitz setting.

Based on joint work with Konstantin Pankrashkin.

3 Bifulco, Patrizio

FernUniversität in Hagen

The heat content on compact quantum graphs and a combinatorial formula

Abstract: We introduce the heat content on compact finite metric graphs which is given by the joint L^1 norm of the fundamental solution (induced by the so-called heat kernel) of the heat equation on the product space, thus leading to a time-dependent quantity. We begin by introducing basic properties of the heat content which are already known on domains and manifolds. Using the so-called *path-sum-formula* for the heat kernel on metric graphs due to Roth, we derive a combinatorial expansion for the heat content leading to a small-time expansion for the heat content. If time permits, we discuss whether a Faber-Krahn-inequality may hold for any time $t > 0$ similar as for the first eigenvalue or the *torsional rigidity* of a compact finite metric graph.

This talk will be based on joint work with Delio Mugnolo and Matthias Täufer (both from Hagen).

4 Farhat, Shahnaz

Constructor University Bremen

Expansion of the Many-body Quantum Gibbs State of the Bose-Hubbard Model on a Finite Graph

In this talk, we consider the many-body quantum Gibbs state for the Bose-Hubbard model on a finite graph at positive temperature. We scale the interaction with the inverse temperature, corresponding to a mean-field limit where the temperature is of the order of the average particle number. For this model it is known that the many-body Gibbs state converges, as temperature goes to infinity, to the Gibbs measure of a discrete nonlinear Schrödinger equation, i.e., a Gibbs measure defined in terms of a one-body theory. In this work we extend these results by proving an expansion to any order of the many-body Gibbs state with inverse temperature as a small parameter. The coefficients in the expansion can be calculated as vacuum expectation values using a recursive formula.

5 Fischer, Florian

IAM, University of Bonn

p-Parabolic Graphs

The type problem, i.e., to decide whether a graph is of p-parabolic or p-hyperbolic type, is a classical topic of discrete quasi-linear potential theory. In this talk, we will show some characterisations for p-parabolicity in the setting of locally summable graphs, and if the time permits, we also have a look at the extremal cases $p = 1$ and $p = \infty$. The talk shows work in progress with Andrea Adriani and Alberto Setti.

6 Karabash, Illia

IAM, University of Bonn

Continuity of eigenvalues under the G-convergence and homogenization of dissipative wave equations.

The homogenization of eigenvalues of nonselfadjoint partial differential operators is not adequately studied yet. We consider this problem on the example of a Maxwell system with dissipative boundary conditions and show also some applications. The talk is based on joint work with Matthias Eller.

7 Kennedy, James

University of Lisbon

Optimising the fundamental gap of a quantum graph

The gap conjecture, proved about 15 years ago by Andrews and Clutterbuck, asserts that the fundamental eigenvalue gap of a Schrödinger operator on a convex domain of fixed diameter with a convex potential, is minimised in the degenerate limit by the Schrödinger operator on an interval of the same diameter, with constant potential. This generalises a roughly 30-year-old result of Lavine, for Schrödinger operators with convex potentials on intervals.

Here we explore what kinds of results can hold in the setting of Schrödinger operators on compact metric trees (the most natural graph analogue of convex domains, where convex potentials can be defined in a natural way). We show that, in general, lower bounds analogous to those on domains are not possible.

More precisely, if one fixes the diameter of the graphs and an upper bound on the L^∞-norm of the potentials, then there is still a sequence of graphs whose fundamental gap converges to zero; while even on a fixed graph, one can find a sequence of convex potentials (whose L^∞-norm explodes) such that the fundamental gap converges to zero.

However, on a given graph, if one restricts to potentials whose L^∞-norm satisfies an a priori bound, then general compactness results based on Helly's theorem allow one to recover minimising and maximising convex potentials. In this case, the minimisers will be piecewise linear, but not constant in general.

In fact, the constant potential being a minimiser seems to be a "rare" property in some sense, which we will try to make more precise in the talk.

This is based on joint work with Mohammed Ahrami, Zakaria El Allali, and Evans Harrell.

8 Naderi, Kiyan

Universität Oldenburg

An embedded trace theorem for infinite metric trees with applications to transmission problems with mixed dimensions

For a class of weighted infinite metric trees we propose a definition of the boundary trace which maps H^1-functions on the tree to L^2-functions on a compact Riemannian manifold. For a range of parameters, the precise Sobolev regularity of the traces is determined. This allows one to show the well-posedness for a Laplace-type equation on infinite trees interacting with Euclidean domains through the boundary. Based on joint works with Valentina Franceschi (Padova), Maryna Kachanovska (Paris) and Konstantin Pankrashkin (Oldenburg).

9 Périce, Denis

Constructor University Bremen

Gyrokinetic limit of the 2D Hartree equation in a large magnetic field

We study the dynamics of two-dimensional interacting fermions submitted to a homogeneous transverse magnetic field. We consider a large magnetic field regime, with the gap between Landau levels set to the same order as that of potential energy contributions. Within the mean-field approximation, i.e. starting from Hartree's equation for the first reduced density matrix, we derive a drift equation for the particle density. We use vortex coherent states and the associated Husimi function to define a semi-classical density almost satisfying the limiting equation. We then deduce convergence of the density of the true Hartree solution by a Dobrushin-type stability estimate for the limiting equation.

10 Rehmeier, Marco

Bielefeld University

Nonlinear Fokker–Planck–Kolmogorov equations as gradient flows on the space of probability measures

We propose a general method to identify nonlinear Fokker–Planck–Kolmogorov equations (FPK equations) as gradient flows on the space of probability measures on $\mathbb{R}^d$ with a natural differential geometry. Our notion of gradient flow does not depend on any underlying metric structure such as the Wasserstein distance, but is derived from purely differential geometric principles. We explicitly

identify the associated energy functions E and show that these are Lyapunov functions for the FPK solutions. These results cover classical and generalized porous media equations, where the latter have a generalized diffusivity function and a nonlinear transport-type first-order perturbation. Joint work with Michael Röckner.

11 Rose, Christian

Technische Universität Hamburg-Harburg

Sobolev dimensions and heat kernels on graphs

On Riemannian manifolds it is known that Sobolev inequalities in balls are equivalent to the conjunction of Gaussian upper bounds and volume doubling. On graphs, this equivalence is expected to hold on large balls. In the case of the normalizing measure, i.e., bounded Laplacian, it turns out that an additional regularity condition on the measure is necessary for the equivalence to hold. If a generalization and unification to arbitrary measure, i.e., possibly unbounded Laplacians, is desired, a new local regularity condition enters the equivalence naturally. Moreover, the dimension of Sobolev inequalities in balls has to vary and depends on the maximal vertex degree inside the ball. It converges if the vertex degree does not grow too fast. This is joint work with Matthias Keller.

12 Schmeckpeper, Dennis

Technische Universität Hamburg-Harburg

What actually are periodic metric graphs?

$\mathbb{Z}^d$-periodic metric graphs are well-studied structures in the quantum graph community which comes with a lot of folklore knowledge. In this talk we propose an abstract, self-contained definition for general Γ-periodic metric graphs for some suitable topological group Γ. We give sufficient and some necessary conditions for the existence of a compact fundamental domain and give an explicit description via so-called Dirichlet domains. We can then *recover* often made (implicit) assumptions on the properties of said fundamental domains.

13 Schonlau, Liza

University of Bonn

Computation of the DOS of Random Schrödinger Operators with Cauchy Distribution via Supersymmetry.

We consider the Schrödinger operator with random potential V on the lattice $\mathbb{Z}^d$. The density of states (DOS) gives the number of eigenstates per unit energy and per unit volume. Assuming V has positively correlated Cauchy distribution, we will derive an exact formula for the finite-volume, averaged DOS with the

help of supersymmetry. Finally, we extend our setting to a simple case of negatively correlated potential.

14 Sztonyk, Kinga

Wrocław University of Science and Technology

Intrinsic ultracontractivity of Feynman-Kac semigroups for cylindrical stable processes

We consider the Schrödinger operator

$$K = K_0 + V,$$

where

$$K_0 = \sqrt{-\frac{\partial^2}{\partial x_1^2}} + \sqrt{-\frac{\partial^2}{\partial x_2^2}}.$$

This is an example of a nonlocal, anisotropic, singular Lévy operator. We study potentials $V : \mathbb{R}^2 \to \mathbb{R}$ such that $V(x)$ goes to infinity as $|x| \to \infty$. The operator $-K_0$ is a generator of a process $X_t = (X_t^{(1)}, X_t^{(2)})$, sometimes called cylindrical, such that $X_1^{(1)}$, $X_2^{(2)}$ are independent symmetric Cauchy processes in $\mathbb{R}$.

The Schrödinger operator K generates the Feynman-Kac semigroup of operators

$$T_t f(x) = E^x \left(\exp\left(-\int_0^t V(X_s)\, ds \right) f(X_t) \right).$$

Operators T_t are compact for every $t > 0$. There exists an orthonormal basis $\{\phi_n\}_{n=1}^{\infty}$ in $L^2(\mathbb{R}^2)$ and a corresponding sequence of eigenvalues $\{\lambda_n\}_{n=1}^{\infty}$, $0 < \lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \ldots$, $\lim_{n\to\infty} \lambda_n = \infty$ such that $T_t\phi_n = e^{-\lambda_n t}\phi_n$. We can assume that ϕ_1 is positive and continuous on $\mathbb{R}^2$. The main result I would like to present in this talk concerns estimates for ϕ_1 and intrinsic ultracontractivity of the semigroup T_t under certain conditions on the potential V.

15 Yannick, Thomas

Potsdam University

Sturmian Hamiltonians and the Kohmoto Butterfly

The central objects of interest in this talk are Schröndinger operators with potentials given by Sturmian sequences. It is well known that this model for 1d-quasicrystals has spectral defects which lead to discontiunities of the spectral map. In this talk we give a different mathematical viewpoint that resolves these discontiunities and which allows explicit descriptions of the defects. If time permits, we explain how our viewpoint can be useful for further investigations of Sturmian Hamiltonians.

This is based on a joint work with Siegfried Beckus