FernUniversität in Hagen
Fakultät für Mathematik und Informatik

AIG
Artificial Intelligence Group

Creating New Extension-Based Semantics Based On Gradual Semantics in Abstract Argumentation

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vorgelegt von
Carola Katharina Bauer

Erstgutachter: Prof. Dr. Matthias Thimm
Artificial Intelligence Group

Betreuer: Kenneth Skiba
Artificial Intelligence Group

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Zusammenfassung

Ranking-basierte und *extensionsbasierte Semantiken* sind zwei wichtige semantische Familien innerhalb der abstrakten Argumentationstheorie, die sich in Bezug auf die Ziele und der Art des Ergebnisses unterscheiden. Diese Masterarbeit formuliert neue extensionsbasierte Semantiken auf Basis von ranking-basierten Semantiken und kombiniert so die Vorteile beider Ansätze. Die neuen Semantiken werden formal definiert und auf Eigenschaften wie *Zulässigkeit* untersucht.

Abstract

Ranking-based and *extension-based semantics* are two important families of semantics in *abstract argumentation theory* that differ regarding goals and types of outcome. Given an argumentation framework, an *extension-based semantics* returns extensions, i.e., sets of arguments that can be accepted together. However, a detailed evaluation of an argument's strength is missing. In contrast, *ranking-based semantics* focus on evaluating the strength of arguments by assigning values or defining a ranking order. However, the relative strength of an argument allows no conclusion as to which arguments can be accepted together.

This thesis formulates new extension-based semantics based on gradual semantics (a particular type of ranking-based semantics), thus combining the advantages of both approaches. Given an argumentation framework, we use the strength of an argument given by gradual semantics to determine whether an argument is accepted in an extension. Different possibilities regarding the conditions for acceptance and the gradual semantics used are explored. The new semantics are formally defined and evaluated for principles such as *admissibility*.

Contents

1 Introduction

The study of computational argumentation has become essential to AI research. Artificial systems with the ability to argue – i.e., evaluate and exchange arguments and form conclusions – could potentially be used for legal reasoning, medical decision support, political decision-making, or argument-mining tools [10].

Whereas other computational argument models consider the arguments' internal structure or the communicative act of exchanging them, *abstract argumentation theory* proposed by Dung [38] focuses on the attack relationships between arguments as abstract entities in an *argumentation framework*.

Due to its high level of abstraction and potential for generalization, abstract argumentation theory has stimulated a plethora of research since it was first suggested [10]. One area of interest has always been *argumentation semantics*. Argumentation semantics provide systematic methods for the evaluation of arguments. The research literature identifies different approaches, i.e., different families of semantics.

Ranking-based and *extension-based semantics* are two important families of semantics in abstract argumentation theory that are fundamentally different when it comes to goals and types of outcome [1]. Given an argumentation framework, an *extension-based semantics* returns extensions, i.e., sets of arguments that can be accepted together [38]. In contrast, *ranking-based semantics* focus on evaluating the strength of arguments in an argumentation framework by assigning values or defining a ranking order [23]. In *gradual semantics*, a particular type of ranking-based semantics, arguments are given a numerical value representing their strength [7].

Both semantic families have their advantages and disadvantages. With extension-based semantics, sets of arguments that form a valid point of view can be identified. However, a detailed evaluation of an argument's strength is missing [50]. Ranking-based semantics assess the relative strength of each argument by defining a ranking order. However, the relative strength of each argument allows no conclusion as to which arguments can be accepted together [59, 23].

Approach This thesis formulates new *extension-based semantics based on gradual semantics*, thus combining the advantages of both approaches. Given an argumentation framework, we will use the strength of the arguments given by gradual semantics to determine whether an argument is accepted. Different possibilities will be explored regarding the *conditions for acceptance* and the *gradual semantics used*.

In Chapter 2, we will discuss existing ranking- and extension-based semantics. We will introduce principles from existing literature that can be used to evaluate and compare those semantics systematically. We will also briefly examine comparing semantics based on their computational complexity. After laying the theoretical foundations, in Chapter 3, we will analyze related studies that have tried to combine those two semantic families in the past.

The main contribution of this thesis will be to define and evaluate the new *extension-based semantics based on gradual semantics*. In Chapter 4, we will formally define the new semantics and describe the algorithm used to implement them. We will conduct experimental evaluations to determine which gradual semantics and thresholds can be used to ensure that *admissibility* can be guaranteed.

In Chapter 5, we will analyze the newly created extension-based semantics in a principle-based evaluation. We will investigate how the gradual semantics' properties influence the principles fulfilled by the newly created extension semantics. Based on the results, we will conclude this thesis with ideas for future studies in Chapter 6.

2 Theoretical Foundations

Abstract argumentation theory, as suggested by Dung [38], uses abstract, formalized arguments to focus on the interactions between arguments. The internal structure of an individual argument is not analyzed.

An *abstract argumentation framework* (AF) consists of a pair $AF = \langle A, attacks \rangle$. A is defined as a finite set of arguments, $attacks$ as the binary relation on A so that $attacks \subseteq A \times A$. The set of attackers of an argument $a \in A$ is defined as $Att(a) = \{b \in A \mid (b, a) \in attacks\}$. An AF can be depicted as a directed graph (digraph) with arguments represented as nodes and attack relations represented as arrows.

In recent years, there have been various approaches extending Dung's original AFs, e.g., by introducing weighted [49, 48, 32, 5] or bipolar [28, 27] argumentation frameworks. This thesis, however, will focus on non-dynamic, non-weighted classical Dung-style AFs. Non-weighted argumentation frameworks, i.e., AFs in which all arguments have an initial strength of 1, can also be called *flat argumentation graphs* [6].

An *argumentation semantics* is a function σ such that for an $AF = \langle A, attacks \rangle$, $\sigma(AF)$ produces an evaluation of its arguments $a \in A$. Abstract argumentation semantics can be systematically and formally characterized and compared using a *principle-based approach*. Thus, in the following sub-chapters, besides defining extension- and ranking-based semantics, the principles that can be used to evaluate them will be introduced as well [63]. Furthermore, as it is relevant to the usefulness of a given semantics, the computational complexity[1] of the presented extension- and ranking-based semantics will be considered. A lower computational complexity has advantages when using artificial agents for automatic reasoning and makes it easier to understand for humans [45].

Other semantic families besides ranking- and extension-based semantics, such as labelling-based semantics[2] will only be discussed when relevant to concepts of extension- or ranking-based semantics.

2.1 Extension-Based Semantics

Extensions are sets of arguments that can be accepted together and form a coherent point of view [11]. An *extension-based semantics* σ is an abstract argumentation semantics: Given an AF, $\sigma(AF)$ denotes the set of σ-extensions of the AF. If a set $S \subseteq A$ is an extension in $\sigma(AF)$, we say that $S \in \sigma(AF)$.

An argument $a \in A$ can be called *acceptable* with regard to a set S iff for every argument $b \in A$ attacking a, there is an argument in S attacking b and thus defending

[1]Further ideas for other ways of comparing different argumentation semantics can be found in [37], the focus of this thesis, however, will be on computational complexity, and the principles fulfilled.

[2]Labelling-based approaches assign one or multiple labels of a set of predefined labels to each argument, e.g., *in* for accepted, *out* for rejected and *undec* for undecided arguments that cannot be categorized as *in* or *out*. Semantics that are based on the labels *out*, *undec*, and *in* can be turned into extension-based approaches by mapping the labels to extensions [11, 23].

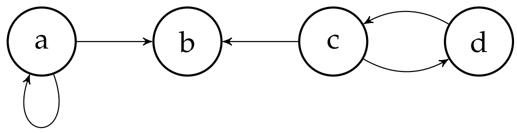

Figure 1: The abstract argumentation framework $AF1$

a. A set of arguments S is *conflict-free* iff there are no arguments $a, b \in S$ such that a attacks b. A set S can be called *admissible* iff it is conflict-free, and each included argument is acceptable with regard to S [11, 38].

Example 1. Regarding $AF1$ in Figure 1, $\{c\}, \{d\}$ and $\emptyset$ are admissible sets.

2.1.1 Classical and Other Extension-Based Semantics

The extension-based semantics originally suggested by Dung [38] include *complete*, *preferred*, *stable* and *grounded* semantics.

Preferred semantics A *preferred* extension E describes the largest possible admissible set in an AF such that every new argument added to E destroys its admissibility. At least one *preferred* extension can be found in every AF.

Example 2. With regard to $AF1$ in Figure 1, $\{c\}$, and $\{d\}$ are preferred extensions.

Stable semantics If each argument in an AF not belonging to a *preferred* extension E is attacked by an argument in E, the preferred extension can be called *stable*. However, not every *preferred* extension can be called *stable*, and not every AF has a stable extension.

Example 3. Regarding $AF1$ in Figure 1, there is no stable extension.

Complete semantics If every possible argument that is acceptable with regard to S is in S and S is *admissible*, then S can be called a *complete* extension. Every *preferred* extension is always a *complete* one.

Example 4. With regard to $AF1$ in Figure 1, $\{c\}$, $\{d\}$ and $\emptyset$ are *complete* extensions.

Grounded semantics *Grounded* extensions are extensions that are minimal *complete* extensions such that every argument in the *grounded* extension is part of each *complete* extension. There can only be one *grounded* extension in an AF, whereas several *preferred*, *stable*, or *complete* extensions are possible. A *grounded extension* E is the least fixed point of the characteristic function $F_{AF} : 2^A \rightarrow 2^A$ of an $AF = \langle A, attacks \rangle$. This function F_{AF} can be characterized as

$$F_{AF}(E) = \{x \in A \mid E \text{ defends } x\}.$$

Example 5. With regard to $AF1$ in Figure 1, $\emptyset$ is the *grounded* extension.

Beyond these semantics suggested by Dung in 1995, several other extension-based approaches have altered or completely neglected Dung's original concept of admissibility.

Prudent semantics *Prudent* semantics, for instance, are based on *p-admissibility*, a stricter form of *acceptability*. An extension E is *p-admissible* iff it is conflict-free, every argument $a \in E$ is defended by E, and there are no $a, b \in E$ such that a indirectly attacks b. An argument a indirectly attacks an argument b iff there is an odd-length path from a to b. Prudent variants of classical semantics include the *p-preferred*, *p-complete*, *p-grounded*, and *p-stable semantics* [63, 29].

Example 6. Concerning $AF1$ in Figure 1, the *grounded*, *preferred*, *stable*, and *complete* extension coincides with the respective *p*-extension.

Non-admissible semantics Non-admissible semantics disregard the concept of admissibility to some extent: Naive-based Semantics are based on the concept of *conflict-freeness* instead, thus neglecting the imperative that an extension needs to defend itself against attacking arguments [12]. Prominent examples include the *naive*, *stage*, *stage*2 and *cf*2 *semantics* [44, 65, 14].

An extension E is a *naive extension* iff it is conflict-free and maximal among the conflict-free sets s.t. for any AF, $E \in mcf(AF)$, i.e., $E \in naive(AF)$ iff $E \in cf(AF)$ and there is no $T \in cf(AF)$ where $E \subset T$ [44].

Example 7. With regard to $AF1$ in Figure 1, $\{c\}$ and $\{b, d\}$ are *naive* extensions.

Weak admissibility semantics *Weak admissibility semantics* reduce the classical notion of admissibility: *Weak admissibility* is based on the underlying idea that an extension E only needs to defend itself against arguments that have a chance of being accepted, realized through the concept of the *reduct* of an extension E (AF^E).

The reduct AF^E describes a reduced argumentation framework of an $AF = \langle A, attacks \rangle$ with all the arguments that are neither attacked by E nor in E ($A' = A \setminus (E \cup \{b \in A \mid E\ attacks\ b\})$, s.t. $AF^E = \langle A', attacks \cap\ A' \times A' \rangle$.

An extension E is *weakly admissible* iff it is conflict-free, and every attacker $y \in A$ of E is not part of a weakly admissible set $ad^w(AF)$ of the E-reduct AF^E, i.e., $y \notin\ \cup ad^w(AF^E)$ [16].

Variants of classical semantics based on *weak admissibility* include *w-preferred*, *w-complete*, *w-grounded* and *w-stable semantics* [63].

Example 8. With regard to $AF1$ in Figure 1, the *complete* extensions are $\{c\}$, $\{d\}$ and $\emptyset$, whereas the weakly complete extension consist of $\{c\}$, $\{b, d\}$ and $\emptyset$.

While there are plenty of other semantics with alternate approaches, those will not be discussed in detail in this thesis due to space limitations.

2.1.2 Computational Complexity

Computational problems can be grouped in basic *complexity classes* according to their computational complexity concerning resources such as time and memory [42].

Polynomial time (P) Problems that can be solved in polynomial time have an algorithm that, for each instance of size $|x|$, produces its answer after at most $|x|^k$ (with k being a fixed constant). A polynomial algorithm is considered to be efficient for sequential algorithms [42].

The classes *NP* and *co-NP* The classes *NP* and *co-NP* describe complexity classes in which a decision problem can be verified in polynomial time.

- ***NP*** A decision problem Q belongs to class *NP* if an actual witness y out of a set of potential witnesses for an instance x ($y \in W(x)$) can be found in polynomial time ($\exists y \in W(x)$:$< x, y > \in W_Q$) [42].
- ***co-NP*** A decision problem Q belongs to the class *co-NP* if in polynomial time, it can be proven that there is no witness y for an answer x ($\forall y \in W(x)$: $< x, y > \notin W_Q$) [42].

Hardness and completeness With the concept of *reducibility* [42], the problems of a complexity class C can be defined more precisely. If a decision problem G is C-hard, then it provides efficient methods for solving all of the problems $F \in C$ such that $F \leq_p G$ (meaning it belongs to a class that is at least as hard as C). If G is in C, then G is C-complete.

Polynomial hierarchy The concept of the polynomial hierarchy (PH) [42] is used to structure the relationship between complexity classes: Levels of complexity classes are differentiated: The complexity classes Π_k^P and Σ_k^P each belong to level k. Level 0 is comprised of P. The first level consists of *co-*NP=Π_1^P and NP=Σ_1^P. The second level consists of Π_2^P and Σ_2^P, level k consists of Π_k^P and Σ_k^P.

The polynomial hierarchy consists of all its classes at every level such that

$$PH = \bigcup_{k=0}^{\infty} \sum_{k}^{P} = \bigcup_{k=0}^{\infty} \Pi_k^P.$$

Every class in the polynomial hierarchy is contained in *PSPACE*, with *PSPACE* describing all sets of decision problems that can be solved with a Turing machine in a polynomial amount of space. The hardest problems in *PSPACE* are *PSPACE-complete*.

Computational problems in extension-based semantics The problem of computing all extensions under specific extension-based semantics is important in practice. However, it cannot be reduced to a simple decision problem, and determining its computational complexity is more challenging. Instead, most often, research concerned with the computational complexity of a semantics [42, 39] focuses on the following problems:

Skeptical & credulous acceptance (*Skept*$_\sigma$ & *Cred*$_\sigma$) One popular decision problem regarding an argumentation framework $AF = \langle A, attacks \rangle$ concerns determining the overall acceptance status of a single argument $a \in A$ under a specific extension-based semantics σ [42]:

- An argument $a \in A$ is *credulously accepted* under a semantics σ iff it is contained in at least one extension $E \in \sigma(AF)$.
- An argument $a \in A$ is *skeptically accepted* iff it appears in all extensions $E \in \sigma(AF)$.
- An argument $a \in A$ is *rejected* iff it appears in no extensions $E \in \sigma(AF)$ at all [11, 39].

Verification of an extension (*Ver*$_\sigma$) Another decision problem with regard to an extension-based semantics σ and an argumentation framework $AF = \langle A, attacks \rangle$ is to verify if a given set $S \subseteq A$ is an extension such that $S \in \sigma(AF)$ [42, 39].

Existence of an extension (*Exists*$_\sigma$) Another popular decision problem can be to prove the existence of an extension under an argumentation semantics σ such that $\exists\ E \in \sigma(AF)$. Alternatively, one could also prove the existence of a non-empty extension such that $\exists\ E \in \sigma(AF)$ with $E \neq \emptyset$ [42].

Table 1: Complexity of extension-based argumentation semantics (C-c denotes completeness for class C)

	Problem			
Semantics	*Cred*$_\sigma$	*Skept*$_\sigma$	*Exists*$_\sigma$	*Ver*$_\sigma$
conflict-free	in P	trivial	trivial	in P
naive	in P	P-c	trivial	in P
grounded	P-c	P-c	trivial	P-c
stable	NP-c	co-NP-c	NP-c	in P
complete	NP-c	P-c	trivial	in P
cf2	NP-c	co-NP-c	trivial	co-NP-c
preferred	NP-c	Π_2^P-c	trivial	co-NP-c
stage	$\sum_2^P$-c	Π_2^P-c	trivial	co-NP-c
stage2	$\sum_2^P$-c	Π_2^P-c	?	co-NP-c
grounded^w	PSPACE-c	PSPACE-c	trivial	PSPACE-c
complete^w	PSPACE-c	PSPACE-c	trivial	PSPACE-c
preferred^w	PSPACE-c	PSPACE-c	trivial	PSPACE-c

Overview of the computational complexity of extension-based semantics

Classical (& prudent) semantics Most classical extension-based semantics are of lower computational complexity, except for *preferred* semantics. Coste-Marquis et al. [29] state that *prudent* variants of classical semantics are equally as complex as their Dung-style counterparts.

Grounded semantics As a *grounded* extension can be computed using the characteristic function F_{AF} in polynomial time and there is only one *grounded* extension for every AF, deciding $Cred_\sigma$, $Skept_\sigma$ or Ver_σ is *P-complete*. $Exists_\sigma$ is trivial, as every AF has a *grounded* extension. [42, 38].

Stable semantics As Dimopoulos and Torres [35] have shown, the problem $Exists_\sigma$ is NP-complete for *stable* semantics. However, Ver_σ is in P, as checking if the arguments not in E are all attacked by arguments in E and if E is conflict-free can be done in polynomial time [42]. For $Cred_\sigma$ and $Skept_\sigma$ the computation is *NP-complete* resp. *co-NP-complete* [35].

Complete semantics $Exists_\sigma$ is trivial, as every AF has a *complete* extension [38]. For $Skept_\sigma$ under *complete* semantics, it suffices to check if this argument is in the *grounded* extension (the unique minimal *complete* extension); the problem is thus in P. However, $Cred_\sigma$ is NP-complete. The problem Ver_σ is in P for *complete semantics*, as it is sufficient to check if this extension meets the conditions for conflict-freeness regarding attacks [30, 42].

Preferred semantics $Exists_\sigma$ is trivial, as every AF has a *preferred* extension [38]. $Cred_\sigma$ is *NP-complete* for *preferred* semantics, as it suffices to check if an argument is part of an admissible set. The problem Ver_σ is *co-NP-complete*. However, $Skept_\sigma$ is on the second level of the polynomial hierarchy [35, 42].

Non-admissible & weak admissibility semantics Among non-admissible semantics, *conflict-free* and *naive semantics* prove to be quite low-ranking in the *PH* regarding $Cred_\sigma$, $Skept_\sigma$, $Exists_\sigma$ or Ver_σ. However, *stage* semantics, as well as *stage2* and *cf2* are much more computationally complex. $Cred_\sigma$ as well $Skept_\sigma$ even ranks on the second level of the polynomial hierarchy for *stage2* and *stage semantics* [43, 42].

Due to their recursive nature, semantics based on *weak admissibility* are of an even higher computational complexity, being *PSPACE*-complete for all decision problems except for $Exists_\sigma$ [41].

2.1.3 Evaluation Criteria for Extension-Based Semantics

Besides computational complexity, other criteria for evaluating argumentation semantics have to be considered: In the so-called *principle-based approach*, Baroni and Giacomin, and others [13, 63] have suggested various principles suitable for systematically comparing existing extension-based argumentation semantics (see Table 2).

Language Independence Two AFs $AF_1 = \langle A_1, attacks_1\rangle$ and $AF_2 = \langle A_2, attacks_2\rangle$ are isomorphic iff there is a bijective function $m : AF_1 \rightarrow AF_2$

such that iff $(a, b) \in attacks_1$ then $(m(a), m(b)) \in attacks_2$.

The *language independence* principle as defined in [13] is satisfied by a semantics σ iff for every two isomorphic argumentation frameworks AF_1 and AF_2, equivalent extensions $E \in \sigma(AF)$ are produced such that $\sigma(AF_2) = \{m(E) \mid E \in \sigma(AF_2)\}$.

All extension-based semantics satisfy *language independence*, as the extensions are based on attack relations instead of an argument's underlying properties [11].

Conflict-Freeness The *conflict-freeness* principle as defined in [13] is satisfied by a semantics σ iff for every AF every extension $E \in \sigma(AF)$ is a conflict-free set [13].

Conflict-freeness is satisfied by all extension-based semantics mentioned so far [11] (see Table 2). In *conflict-tolerant semantics*, however, extensions are not necessarily conflict-free [9].

Defense The *defense* principle as defined in [13] is satisfied by a semantics σ iff for every extension $E \in \sigma(AF)$ every argument $a \in E$ is defended by E.

All Dung-style semantics, as well as their prudent variants, fulfill this principle [63]. Naive-based semantics such as *cf2*, *stage*, *stage2*, or *naive semantics* violate it [13, 11]. Weak admissibility semantics also only satisfy a weaker form of *defense* as defined in [16].

Admissibility The *admissibility* principle as defined in [13] is satisfied by a semantics σ iff for every AF, every extension $E \in \sigma(AF)$ is an admissible set. If a semantics σ fulfills the *admissibility* principle, then it also satisfies the *conflict-freeness* and the *defense* principle.

Whereas naive-based and weak admissibility semantics do not satisfy *admissibility*, classical semantics and their prudent variants do [63].

Strong Admissibility The *strong admissibility* principle as defined in [13] is satisfied by a semantics σ iff for every AF every extension $E \in \sigma(AF)$ is admissible and every argument $a \in E$ is strongly defended by E, i.e., each attacker $b \in Att(a)$ is attacked by an argument $c \in E \setminus \{a\}$ with $E \setminus \{a\}$ strongly defending c.

Only *grounded* semantics satisfy *strong admissibility* among the extension-based semantics suggested by Dung [13]. Among the prudent semantics, *p-grounded* semantics satisfies *strong admissibility* as well [63].

Reduct Admissibility To capture the deviating notions of *admissibility* in newer extension-based semantics, altered concepts of *admissibility* were introduced. The *reduct admissibility* principle as defined in [33] is satisfied by a semantics σ iff for every AF every extension $E \in \sigma(AF)$ is conflict-free, and no argument $b \in A$ attacking $a \in E$ is part of a set of the E-reduct regarding σ ($b \notin \bigcup \sigma(AF^E)$).

While weak admissibility semantics such as *w-complete semantics* do not fulfill *admissibility*, *reduct admissibility* is guaranteed. Classical semantics fulfill *reduct admissibility* as well. In contrast, all naive-based semantics do not satisfy this principle [33].

Semi-Qualified Admissibility The *semi-qualified admissibility* principle as defined in [33] is satisfied by a semantics σ iff for every AF every extension $E \in \sigma(AF)$ is conflict-free and every argument $a \in E$ is defended by E against any argument $b \in Att(a)$ which is part of any extension in $\sigma(AF)$ ($b \in \bigcup \sigma(AF)$).

Semi-qualified admissibility is neither satisfied for naive-based nor *weak-admissibility semantics*, whereas classical semantics and their prudent variants fulfill it [33].

I-Maximality The *I-maximality* principle as defined in [13] is satisfied by a semantics σ iff for every $E_1, E_2 \in \sigma(AF)$ given that E_1 contains E_2, then $E_1 = E_2$.

I-maximality is fulfilled by every semantics mentioned so far, except for *complete*, *w-complete* and *p-complete semantics*[13, 63, 33].

Naivety The *naivety* principle is satisfied by a semantics σ iff for every AF every extension $E \in \sigma(AF)$ is a *naive* extension, i.e. is a maximal conflict-free set.

Only *stable* semantics satisfies *naivety* among the extension-based semantics suggested by Dung [13, 11]. Among prudent semantics, it is only fulfilled by the *p-stable* semantics. However, naive-based semantics, i.e., *stage*, *stage2*, *naive* and *cf2* semantics, all satisfy this principle, while the three weak admissible semantics *w-complete*, *w-grounded*, and *w-preferred* do not [63].

Reinstatement The *reinstatement* principle as defined in [13] is satisfied by a semantics σ iff for every AF every extension $E \in \sigma(AF)$ contains every argument it defends.

All Dung-style classical semantics satisfy this principle [13]. Among prudent variants of classical semantics, only *p-stable* semantics fulfills this principle, whereas weak admissibility semantics *w-complete*, *w-grounded*, and *w-preferred* do. All naive-based semantics do not fulfill *reinstatement* [63].

Weak Reinstatement The *weak reinstatement* principle as defined in [13] is satisfied by a semantics σ iff for every $AF = \langle A, attacks \rangle$ every extension $E \in \sigma(AF)$ contains every argument $a \in A$ it strongly defends.

A semantics fulfilling the *reinstatement* principle also fulfills the *weak reinstatement* principle [63]. The naive-based semantics *stage2* and *cf2* do not fulfill *reinstatement*, but fulfill *weak reinstatement*, whereas *naive* and *stage* semantics do fulfill neither[63].

CF-Reinstatement The *CF-reinstatement* principle as defined in [13] is satisfied by a semantics σ iff for every AF every extension $E \in \sigma(AF)$ contains every argument a it defends that does not result in E becoming conflicted ($E \bigcup \{a\} \in cf(AF)$).

CF-Reinstatement is fulfilled for *grounded*, *preferred*, *stable* and *complete* as well as for *p-stable*, *cf2*, *stage*, *stage2*, *naive*, *w-complete*, *w-grounded* and *w-preferred* semantics [16, 13, 43].

Directionality Given an $AF = \langle A, attacks \rangle$, a set $U \subseteq A$ is *unattacked* iff there is no $a \in A \backslash U$ such that a attacks any argument $b \in U$. The *directionality* principle as defined in [13] is satisfied by a semantics σ iff for any AF and any unattacked set $U \subseteq A$, any admissible extension E of U is also admissible in the AF as a whole, i.e., $\sigma(AF, U) = \sigma(AF|_U)$ with $\sigma(AF, U) = \{E \cap U \mid E \in \sigma(AF)\}$.

Stable semantics does not satisfy this principle, whereas *complete, grounded*, and *preferred* semantics do [13]. Weak admissibility semantics do not fulfill this principle, whereas the naive-based semantics *cf2* and *stage2* do [33]. Among prudent variants of classical semantics, only *p-grounded* semantics fulfills this principle [63].

Irrelevance of Necessarily Rejected Arguments (INRA) The property *irrelevance of necessarily rejected arguments*, as suggested by [31], concerns the irrelevance of arguments attacked by every extension: A semantics σ fulfills *INRA*, iff – given an $AF = \langle A, attacks \rangle$ and an argument $a \in A$ that is attacked by every extension $E \in \sigma(AF)$ – deleting a does not alter the set of extensions s.t. for the altered framework AF', $\sigma(AF) = \sigma(AF')$.

Among the naive-based semantics mentioned above, *INRA* is only fulfilled by the *naive* semantics; for classical semantics, only *grounded* and *complete* semantics satisfy *INRA*. The property has not been evaluated for prudent and weakly admissible semantics yet.

Modularization The *modularization* principle as defined in [16] is satisfied by a semantics σ iff for any $AF = \langle A, attacks \rangle$ with an extension $E \in \sigma(AF)$ and an extension E' with regard to the reduct AF^E ($E' \in \sigma(AF^E)$) – $E' \cup E \in \sigma(AF)$.

All Dung-style semantics, as well as their weak-admissibility-variants, satisfy *modularization*, whereas *naive* semantics do not satisfy it [16]. This property has yet to be investigated for *prudent* variants and other naive-based semantics.

SCC Recursiveness *Strongly Connected Components (SCCs)* of an $AF = \langle A, attacks \rangle$ describe the equivalence classes for path-equivalent arguments in an AF. *Path-equivalence* PE_{AF} between arguments can be characterized as:

- $\forall a \in A$, $(a, a) \in PE_{AF}$
- for distinct arguments: $\forall a, b \in A, (a, b) \in PE_{AF}$ iff a, b are connected by paths from a to b and b to a

The *SCC recursiveness* principle as defined in [14] is satisfied by a semantics σ iff a *SCC recursive scheme*, i.e., a selection function GF_{BF} can be identified for any AF with which all extensions can be constructed in an incremental algorithm:

1. All $SCCs$ of an AF, denoted as $SCCS_{AF}$, are identified and evaluated according to their respective dependencies.

   Given an SCC $S \in SCCS_{AF}$, the $SCCs$ directly attacking S are called parents of S and described more formally as $sccpar_{AF}(S) = \{P \in SCCS_{AF} \mid P \neq S,\ and\ P\ attacks\ S\}$.

   The $SCCS_{AF}$ are partially ordered according to their attack relations. Iff $sccpar_{AF}(S) = \emptyset$, then S is initial.

2. Starting from the initial $SCCs$, partial extensions are identified by applying a *base function* BF, i.e. a specific extension-based semantics, to a single SCC as an AF.

3. Nodes directly attacked by the partial extensions within subsequent $SCCs$ are suppressed, following the principle of *directionality*.

4. The previous steps (1.-3.) are applied recursively to all subsequent, modified $SCCs$.

According to Baroni [14], every admissibility-based Dung-Style semantics fulfills the *SCC recursiveness* principle. If an argumentation semantics is *SCC-recursive* and prescribes that there is a set of nonempty extensions, it fulfills *directionality* as well [14]. Among prudent variants of classical semantics, however, this principle is not fulfilled [63]. Among weak admissibility semantics, only for *w-preferred* semantics *SCC recursiveness* has been proven [41]. Among naive-based semantics, only *cf2* and *stage2* satisfy this principle [33].

Table 2: Extension-based semantics analyzed with respect to principles according to [33, 41, 16, 63, 13, 43]

Principles	Semantics: co	gr	pr	st	w-co	w-gr	w-pr	naive	cf2	stg	stg2	p-co	p-gr	p-pr	p-st
Admissibility	✓	✓	✓	✓	×	×	×	×	×	×	×	✓	✓	✓	✓
Strong Adm.	×	✓	×	×	×	×	×	×	×	×	×	×	✓	×	×
Reduct Adm.	✓	✓	✓	✓	✓	✓	✓	×	×	×	×	✓	✓	✓	✓
Semi-Qual.Adm.	✓	✓	✓	✓	×	×	×	×	×	×	×	✓	✓	✓	✓
Conflict-Freeness	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Defense	✓	✓	✓	✓	×	×	×	×	×	×	×	✓	✓	✓	✓
Naivety	×	×	×	✓	×	×	×	✓	✓	✓	✓	×	×	×	✓
I-Maximality	×	✓	✓	✓	×	✓	✓	✓	✓	✓	✓	×	✓	✓	✓
Reinstatement	✓	✓	✓	✓	✓	✓	✓	×	×	×	×	×	×	×	✓
Weak Reinst.	✓	✓	✓	✓	✓	✓	✓	×	✓	×	✓	×	×	×	✓
CF-Reinst.	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	×	×	✓
Directionality	✓	✓	✓	×	×	×	×	×	✓	×	✓	×	✓	×	×
Modularization	✓	✓	✓	✓	✓	✓	✓	×	?	?	?	?	?	?	?
SCC recursiveness	✓	✓	✓	✓	**?**	**?**	✓	×	✓	×	✓	×	×	×	×
INRA	✓	✓	×	×	**?**	**?**	**?**	✓	×	×	×	**?**	**?**	**?**	**?**

On the evaluation of selected principles As Dvořák et al. [45] have stated, some of the principles mentioned, such as *directionality* and *SCC recursiveness* prove to be more important than others when evaluating semantics, as the fulfillment of both by an argumentation semantics makes it possible to compute extensions incrementally, following the partial order of the *SCCs*.

Basic principles, however, such as the principle of *language independence*, can be neglected, as the abstract nature of the arguments in an AF prevents any other argument properties from being considered for the semantics.

Other principles like *allowing abstention* have been discussed in [17]. However, they will not be considered due to space limitations [11].

2.2 Ranking-Based Semantics

Whereas extension-based semantics aim to deliver sets of arguments that can be accepted together, the approach of *ranking-based semantics* is slightly different. Given an $AF = \langle A, attacks \rangle$, a *ranking-based semantics* $\sigma(AF)$ is a function that transforms any AF into a ranking $\succeq^{\sigma}_{AF}$ of its arguments [2, 21, 24].

The arguments $a, b \in A$ are ranked according to strength resp. level of relative acceptability such that $a \succeq^{\sigma}_{AF} b$ means that a is at least as acceptable as b, $a \succ^{\sigma}_{AF} b$ means that a is more acceptable than b and $a \simeq^{\sigma}_{AF} b$ means that a and b are equally acceptable.

In *gradual semantics* (first suggested by [26]), arguments are given a numerical value representing their strength by a gradual function S. Given an $AF = \langle A,attacks \rangle$, S assigns a weighting Deg^{S}_{AF} on A such that for any $a \in A$, $Deg^{S}_{AF}(a)$ represents the strength of a [7]. Gradual semantics can be regarded as a subtype of ranking-based semantics [20, 54]. By comparing the argument values, any gradual semantics σ can transform an AF into a ranking $\succeq^{\sigma}_{AF}$ of its arguments [1].

However, there are other ranking-based semantics as well: So-called *pure ranking-based semantics* do not assign a value to each argument. They only define a pre-order, i.e., a preference relationship between the arguments of an AF [1].

2.2.1 Existing Ranking-Based Semantics

Since its first suggestion [2], there have been many instances of ranking-based semantics. Examples of pure ranking-based semantics are the *propagation semantics* suggested by Bonzon et al. [22] or the ranking-based semantics based on subgraphs analysis by Dondio [36]. This thesis, however, will focus on the following gradual semantics:

h-Categorizer (hCat) For the *h-Categorizer* (*hCat*) semantics, Besnard and Hunter [18] have proposed assigning values to arguments based on the value of their direct attackers.

Given an argumentation framework $AF = \langle A, attacks \rangle$ with $a \in A$ and $Att(a) = \{b \in A \mid (b, a) \in attacks\}$ as the set of direct attackers of a, the value $Deg_{AF}^{hCat}(a)$ is determined by the semantic categorizer function:

$$Deg_{AF}^{hCat}(a) = \frac{1}{1 + \sum_{b \in Att(a)} Deg_{AF}^{hCat}(b)}$$

If a has no directly attacking arguments, then $Deg_{AF}^{hCat}(a) = 1$. The *ranking-based categorizer* translates the computed values into a ranking $\succeq_{AF}^{hCat}$ such that $\forall a, b \in AF$, $a \succeq_{AF}^{hCat} b$ iff $Deg_{AF}^{hCat}(a) \geq Deg_{AF}^{hCat}(b)$.

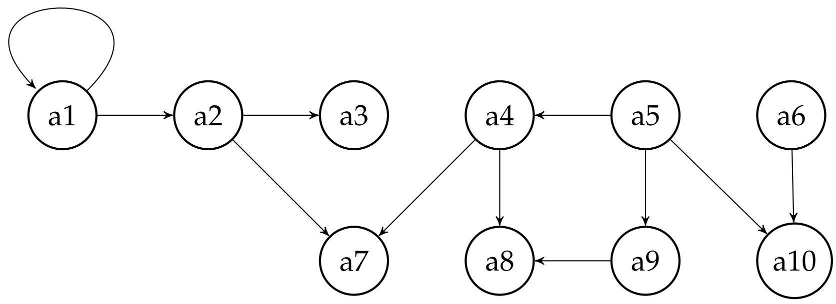

Figure 2: The abstract argumentation framework AF2

There have been several attempts to improve the *h-categorizer*. Being originally proposed for non-weighted, acyclic graph structures [18], Amgoud et al. [5] have extended the *h-categorizer* for use on weighted graphs.

Pu et al. [57] introduced a fixed-point technique to enable the categorizer function to deal with cycles in argumentation graphs, in which Deg_{AF}^{hCat} for an argument $a \in A$ is computed iteratively for k steps until the change to the approximate solution v^k is under a given tolerance ϵ – s.t. $\|v^k - v^{k-1}\| < \epsilon$.

Example 9. For AF2 in Figure 2, the argument values given by the *h-categorizer* semantics with $\epsilon = 0.0001$ are:

- $Deg_{AF}^{hCat}(a1) =Deg_{AF}^{hCat}(a2) = Deg_{AF}^{hCat}(a3) \approx 0.618$,
- $Deg_{AF}^{hCat}(a5)= Deg_{AF}^{hCat}(a6) = 1$,
- $Deg_{AF}^{hCat}(a4)= Deg_{AF}^{hCat}(a8)= Deg_{AF}^{hCat}(a9) = 0.5$,
- $Deg_{AF}^{hCat}(a10) =\frac{1}{3}$,
- and $Deg_{AF}^{hCat}(a7) \approx 0.472$.

This results in the ranking $\succeq_{AF}^{hCat}$:

$a5 \simeq a6 \succ a3 \simeq a2 \simeq a1 \succ a4 \simeq a8 \simeq a9 \succ a7 \succ a10$.

No self-Attack h-Categorizer (nsa) Beuselinck et al. [19] have created the *no self-Attack h-Categorizer (nsa)* semantics based on the approach by Besnard and

Hunter, in which the *h-Categorizer* is modified. The gradual function Deg_{AF}^{nsa} of an argument a is computed iteratively, s.t.

$$Deg_{AF}^{nsa}(a) = \begin{cases} 0 \text{ iff } (a,a) \in attacks \\ \frac{1}{1+\sum_{b \in Att(a)} Deg_{AF}^{nsa}(b)} \; otherwise \end{cases}$$

Thus, the impact of self-attacking arguments is reduced to 0, similar to *extension-based semantics* where self-attacking arguments are predominantly rejected. Like the *hCat* semantics, the *nsa* semantics can deal with cycles in argumentation graphs by using the fixed point technique of Pu et al. [57].

Example 10. For AF2 in Figure 2, the argument values given by the *nsa semantics* for $\epsilon = 0.0001$ are

- $Deg_{AF}^{nsa}(a1)$= $Deg_{AF}^{nsa} = 0$,
- $Deg_{AF}^{nsa}(a2) = Deg_{AF}^{nsa}(a5)$= $Deg_{AF}^{nsa}(a6) = 1$,
- $Deg_{AF}^{nsa}(a3)$= $Deg_{AF}^{nsa}(a4)$= $Deg_{AF}^{nsa}(a8)$= $Deg_{AF}^{nsa}(a9)$= 0.5,
- $Deg_{AF}^{nsa}(a10) = \frac{1}{3}$,
- and $Deg_{AF}^{nsa}(a7) = 0.4$.

This results in the ranking $\succeq_{AF}^{nsa}$:

$a5 \simeq a6 \simeq a2 \succ a3 \simeq a1 \succ a4 \simeq a8 \simeq a9 \succ a7 \succ a10$.

(Euler) Max-based semantics ((E)Mbs) Given an argumentation framework $AF = \langle A, attacks \rangle$ with $a \in A$ and $Att(a) = \{b \in A \,|(b,a) \in attacks\}$, with *Max-based semantics* (Mbs) [5], the quality of attacks is more relevant than their quantity: Originally devised for weighted AFs, the acceptance degree Deg_{AF}^{Mbs} of every argument $a \in A$ is determined for flat graphs by considering the weight of its strongest attacker:

$$Deg_{AF}^{Mbs}(a) = \frac{1}{1 + max_{b \in Att(a)} Deg_{AF}^{Mbs}(b)}$$

Similar to *Mbs*, the *Euler-Max-based semantics* (Embs) also emphasizes the quality of attacks instead of their quantity [7, 6]: Like with *Mbs*, only the strongest attacker is considered for the acceptance degree Deg_{AF}^{Embs}:

$$Deg_{AF}^{Embs}(a) = e^{-\max_{b \in Att(a)} Deg_{AF}^{Embs}(b)}$$

With both *Embs* and *Mbs*, if the graph is acyclic, the results can be computed directly by starting with the unattacked arguments. In the case of cyclic graphs, the values have to be calculated iteratively in k steps with the help of the respective function [6]. Just like with *hCat*, the fixed point technique as suggested by [57] can be used until the change to the approximate solution v^k is under a given tolerance ϵ – s.t. $\|v^k - v^{k-1}\| < \epsilon$. This is possible, as both semantics are continuous and have a unique fixed point [53].

Example 11. For AF2 in Figure 2, the argument values given by the *Mbs* semantics with $\epsilon = 0.0001$ are:

- $Deg_{AF}^{Mbs}(a5)$= $Deg_{AF}^{Mbs}(a6) = 1$,
- $Deg_{AF}^{Mbs}(a8) = \frac{2}{3}$,
- $Deg_{AF}^{Mbs}(a3)$= $Deg_{AF}^{Mbs}(a7)$= $Deg_{AF}^{Mbs}(a2)$= $Deg_{AF}^{Mbs}(a1) \approx 0.618$,
- and $Deg_{AF}^{Mbs}(a10)$= $Deg_{AF}^{Mbs}(a4)$= $Deg_{AF}^{Mbs}(a9) = 0.5$.

For AF2 in Figure 2, the argument values given by the *Embs* semantics with $\epsilon = 0.0001$ are:

- $Deg_{AF}^{Embs}(a5)$= $Deg_{AF}^{Embs}(a6) = 1$,
- $Deg_{AF}^{Embs}(a8) \approx 0.692$,
- $Deg_{AF}^{Embs}(a3) = Deg_{AF}^{Embs}(a7)$= $Deg_{AF}^{Embs}(a2)$= $Deg_{AF}^{Embs}(a1) \approx 0.5672$,
- and $Deg_{AF}^{Embs}(a10)$= $Deg_{AF}^{Embs}(a4)$= $Deg_{AF}^{Embs}(a9) \approx 0.3679$.

This results in the following ranking $\succeq_{AF}^{Mbs}$ resp. $\succeq_{AF}^{Embs}$:

$a5 \simeq a6 \succ a8 \succ a3 \simeq a7 \simeq a2 \simeq a1 \succ a4 \simeq a10 \simeq a9$.

Trust-based semantics (Tbs) The *Trust-based semantics (Tbs)* as devised by da Costa Pereira et al. [32] was initially designed for weighted argumentation frameworks but modified for flat argumentation graphs. Given an $AF = \langle A, attacks \rangle$ and $Att(a) = \{b \in A \mid (b, a) \in attacks\}$, the trust-worthiness Deg_{AF}^{Tbs} of an argument $a \in A$ is computed by considering the strongest (most reliable) attacker in different steps $i \in \{0, 1...\}$.

$$Deg_{AF}^{Tbs} = \lim_{i \to +\infty} f_i(a) \; where$$

$$f_i(a) = \frac{1}{2} f_{i-1}(a) + \frac{1}{2} \, min[1, 1 - max_{b \in Att(a)} f_{i-1}(b)]$$

To determine $\lim_{i \to +\infty} f_i(a)$, the fixed point technique as suggested by [57] can be used as well.

Example 12. For AF2 in Figure 2, the argument values given by the *Tbs* semantics with $\epsilon = 0.0001$ are:

- $Deg_{AF}^{Tbs}(a5)$= $Deg_{AF}^{Tbs}(a6)$= $Deg_{AF}^{Tbs}(a8) = 1$,
- $Deg_{AF}^{Tbs}(a8) \approx 0.9999$
- $Deg_{AF}^{Tbs}(a3)$= $Deg_{AF}^{Tbs}(a7)$= $Deg_{AF}^{Tbs}(a2)$= $Deg_{AF}^{Tbs}(a1) = 0.5$,
- and $Deg_{AF}^{Tbs}(a10)$= $Deg_{AF}^{Tbs}(a4)$= $Deg_{AF}^{Tbs}(a9) = 0 \approx 0.000004$.

This results in the following ranking $\succeq_{AF}^{Tbs}$:

$a5 \simeq a6 \succ a8 \succ a3 \simeq a7 \simeq a2 \simeq a1 \succ a4 \simeq a10 \simeq a9$.

Iterative schema semantics (ITS) The *Iterative schema semantics* (ITS) has been introduced by Gabbay and Rodrigues in [46]. Given an $AF = \langle A, attacks\rangle$ and $Att(a) = \{b \in A \mid (b,a) \in attacks\}$, *ITS* can be used to compute the argument value Deg_{AF}^{ITS} of an argument $a \in A$ iteratively, in different steps $i \in \{0, 1...\}$. Like with *Mbs* or *Tbs*, the value depends on the value of the strongest attacker $b \in A$ in the previous step.

$$Deg_{AF}^{ITS} = \lim_{i \to +\infty} f_i(a) \text{ where}$$

$$\begin{aligned} f_i(a) = (1 - f_{i-1}(a))\; min[\frac{1}{2}, 1 - max_{b \in Att(a)} f_{i-1}(b)] \\ + f_{i-1}(a)\; max[\frac{1}{2}, 1 - max_{b \in Att(a)} f_{i-1}(b)] \end{aligned}$$

For all $a \in A$, $f_0(a)$ is defined as $w(a)$ resp. $w(a) = 1$ for flat graphs. To determine $\lim_{i \to +\infty} f_i(a)$, we will also use the fixed point technique as suggested by [57] with $\epsilon = 0.0001$.

Example 13. For AF2 in Figure 2, the argument values given by the *ITS* semantics are:

- $Deg_{AF}^{ITS}(a5)$= $Deg_{AF}^{ITS}(a6) = Deg_{AF}^{ITS}(a8) = 1$,
- $Deg_{AF}^{ITS}(a8) \approx 0.9999$,
- $Deg_{AF}^{ITS}(a3)$= $Deg_{AF}^{ITS}(a7)$= $Deg_{AF}^{ITS}(a2)$= $Deg_{AF}^{ITS}(a1) = 0.5$,
- and $Deg_{AF}^{ITS}(a10)$= $Deg_{AF}^{ITS}(a4)$= $Deg_{AF}^{ITS}(a9) = 0 \approx 0.000002$.

This results in the following ranking $\succeq_{AF}^{ITS}$:

$a5 \simeq a6 \succ a8 \succ a3 \simeq a7 \simeq a2 \simeq a1 \succ a4 \simeq a10 \simeq a9$.

Counting semantics (Count) The *Counting semantics (Count)* has been introduced by Pu et al. [58, 56]. In contrast to *Mbs*, *ITS* or *Tbs*, not the strongest attacker, but the overall numbers of defenders and attackers are considered. Given an $AF = \langle A, attacks\rangle$, an argument $a \in A$ is more acceptable if the number of defenders is higher and the number of attackers is lower.

The column vector v containing the strength $v(a)$ of every argument $a \in A$ is computed for each walk length k until it converges, s.t.

$$v_\alpha = \lim_{k \to \infty} v_\alpha^{(k)}$$

A damping factor $\alpha \in (0, 1)$ is used to differentiate between different walk lengths for defenders and attackers: If α is higher, more attackers and defenders are considered. However, the computation is slower. Pu et al. [56] recommend a value of α in $[0.9, 0.98]$.

The number of attackers is subtracted, or the number of defenders is added for each walk length k, depending on whether k is odd or even. The sum of defenders or attackers on a walk length k is determined with the help of the attack matrix M[3] and a column vector e of all ones. The attack matrix M is additionally normalized as $\tilde{M} = M/N$ with N being the normalization factor[4] to deal with cyclic graphs. For each walk length k, the counting values v are computed with the following formula:

$$v_\alpha^{(k)} = e - \alpha \tilde{M} v_\alpha^{(k-1)}$$

$$\text{with } v_\alpha^{(0)} = e.$$

To prevent the iterations from being endless in the case of cyclic graphs, the computation is terminated if the difference to the value of the previous walk length is under or equal to a given tolerance ϵ s.t. $\|v_\alpha^{(k)} - v_\alpha^{(k-1)}\| < \epsilon$.

Example 14. For AF2 in Figure 2 with $\epsilon = 0.0001$ and $\alpha = 0.9$, the argument values given by the *Count* semantics are:

- $Deg_{AF}^{Count}(a5)$= $Deg_{AF}^{Count}(a6) = 1$,
- $Deg_{AF}^{Count}(a1)$= $Deg_{AF}^{Count}(a2)$= $Deg_{AF}^{Count}(a3) \approx 0.6896$,
- $Deg_{AF}^{Count}(a9)$= $Deg_{AF}^{Count}(a4) \approx 0.55$,
- $Deg_{AF}^{Count}(a8) \approx 0.505$,
- $Deg_{AF}^{Count}(a7) \approx 0.442$,
- and $Deg_{AF}^{Count}(a10) \approx 0.1$.

This results in the following ranking $\succeq_{AF}^{Count}$:

$a5 \simeq a6 \succ a1 \simeq a2 \simeq a3 \succ a9 \simeq a4 \succ a8 \succ a7 \succ a10$.

M&T Given an $AF = \langle A, attacks \rangle$, an argument $a \in A$, the ranking-based semantics *M&T* by Matt and Toni [52] uses game theoretic notions to compute a concrete value $s_{AF}(a)$ for the strength of every argument. The strength $s_{AF}(a)$ is in [0,1] and determined in a repeated strategic game (AF, a) between two players (opponent and proponent) with imperfect information[5].

The set of possible strategies of the proponent concerning an argument a is defined as $S_P(a) = \{P \mid P \subseteq A, a \in P\}$, the possible strategies of the opponent are defined as $S_O(a) = \{O \mid O \subseteq A\}$.

[3] An attack matrix of an AF is an $n \times n$ Matrix, in which $a_{ij} = 1$ if $(x_j, x_i) \in attacks$ and 0 otherwise.
[4] The infinity norm of the attack matrix is used as a normalization factor [56].
[5] In a game with imperfect information, the two players do not have information on the strategy of their opponent.

The degree of acceptability ϕ of P with respect to O is defined by considering the sum of attacking arguments for O by P ($0_{AF}^{\leftarrow P} = \{(o,p) \in P \times O \mid (o,p) \in attacks\}$) and vice versa ($P_{AF}^{\leftarrow O} = \{(p,o) \in O \times P \mid (p,o) \in attacks\}$) s.t.:

$$\phi(P,O) = \frac{1}{2}(1 + f(|O_{AF}^{\leftarrow P}|) - f(|P_{AF}^{\leftarrow O}|)$$

$$\text{with } f(n) = 1 - \frac{1}{n+1}$$

As it is a zero-sum game, the reward $r_{AF}(P,O)$ of a proponent is equal to the loss of an opponent and has to be paid to the proponent by the opponent.

$$r_{AF}(P,O) = \begin{cases} 0 \text{ if P is not conflict-free} \\ 1 \text{ if P is conflict-free and not attacked by 0} \\ \phi(P,O) \text{ otherwise} \end{cases}$$

The possible strategies are determined through probability distributions p with length $m = |S_P|$and o with length $m = |S_O|$. The probability of the opponent choosing a strategy j is denoted by $o_j \in o$, and the probability of the proponent choosing a strategy i is denoted by $p_i \in p$. The expected payoff E for each argument a as a component can be computed by the following formula:

$$E(a,p,q) = \sum_{j=1}^{n}\sum_{i=1}^{m} p_i o_j r_{i,j}$$

with $r_{i,j} = r_{AF}(P_i, O_j)$ being the reward of the i^{th} strategy for P and the j^{th} strategy for O. The actual score $Deg_{AF}^{M\&T}$ of each argument $a \in A$ as a proponent then can be computed by considering the minimum of the probability distributions available to the opponent ($\underset{q}{min}$) and the maximum of the probability distributions available to the proponent ($\underset{p}{max}$):

$$Deg_{AF}^{M\&T}(a) = \underset{p}{max}\ \underset{q}{min}\ E(a,p,q) = \underset{q}{min}\ \underset{p}{max}\ E(a,p,q)$$

Example 15. For AF2 in Figure 2, the argument values given by the *M&T* semantics are:

- $Deg_{AF}^{M\&T}(a5)$= $Deg_{AF}^{M\&T}(a6) = 1$,
- $Deg_{AF}^{M\&T}(a8) = 0.5$,
- $Deg_{AF}^{M\&T}(a2) = Deg_{AF}^{M\&T}(a3) = Deg_{AF}^{M\&T}(a4)$
  $= Deg_{AF}^{M\&T}(a7) = Deg_{AF}^{M\&T}(a9) = 0.25$,
- $Deg_{AF}^{M\&T}(a10) \approx 0.167$,
- and $Deg_{AF}^{M\&T}(a1) = 0.0$.

This results in the following ranking $\succeq_{AF}^{M\&T}$:

$a5 \simeq a6 \succ a8 \succ a2 \simeq a3 \simeq a9 \simeq a4 \simeq a7 \succ a10 \succ a1$.

2.2.2 Computational Complexity

Whereas there are many studies about the computational complexity of extension-based semantics, ranking-based semantics still need to be addressed.

Amgoud et al. [8] investigated the performance of *weighted max-based* and *weighted h-categorizer semantics* for weighted graphs by evaluating the number of iterations and time needed to calculate the strength of all arguments. They found that *h-categorizer* is the slowest among the two. However, all semantics evaluated scaled well for larger graphs with 100 000 arguments with the number of iterations remaining constantly under 20.

Beuselinck et al. [19] compared the execution time for computing the strength of arguments under *nsa*, *M&T*, and *h-categorizer semantics* and found that the *M&T* semantics explodes in time and systematically reaches a timeout if the argumentation graph has more than 15 arguments. In contrast, *nsa* and *h-categorizer semantics* have low execution times even for larger AFs with 500 arguments s.t. computation takes between 1 and 2 seconds.

Oren et al. [53] have proven that *weighted h-categorizer semantics*, and *weighted max-based semantics*, as well as other comparable semantics, definitely converge, i.e., a unique fixed-point exists.

2.2.3 Evaluation Criteria for Ranking-Based Semantics

Properties suited for evaluating ranking-based semantics are defined in [2, 21, 24, 52]. They can be categorized in *basic general*, *local*, and *global properties* [23].

Basic general properties as defined in [21] concern properties inherent in almost all ranking-based semantics:

Abstraction (Abs) A ranking-based semantics σ satisfies *abstraction* iff for every two isomorphic AFs $AF_1 = \langle A_1, attacks_1 \rangle$ and $AF_2 = \langle A_2, attacks_2 \rangle$ with a bijective function $m : A_1 \rightarrow A_2$, equivalent ranking relations are produced so that $\forall a, b \in A_1$, $a \succeq^{\sigma}_{AF_1} b$ means $m(a) \succeq^{\sigma}_{AF_2} m(b)$ [2].

Independence (In) A ranking-based semantics σ satisfies *independence* iff the ranking position of an argument $a \in A$ in an $AF = \langle A, attacks \rangle$ does not depend on any argument $b \in A$ that is not directly connected to a. Given a *weakly connected component*[6] AF' in an AF, iff $a \succeq^{\sigma}_{AF'} b$, then $a \succeq^{\sigma}_{AF} b$ [2].

Total (Tot) A ranking-based semantics σ satisfies *total* iff any pair of arguments in an AF can be compared [21].

Non-Attacked Equivalence (NaE) Given an $AF = \langle A, attacks \rangle$, with $a, b \in A$, a semantics σ fulfills *NaE* iff all unattacked arguments $a, b \in A$ have the same rank s.t. $a \simeq^{\sigma}_{AF} b$ [21].

[6]Weakly connected components are maximal subgraphs of AF in which all nodes are connected in a path (independent from the direction of the connecting edges).

Local properties as defined in [21] can be used to determine how direct attackers and defenders are treated.

Void Precedence (VP) A ranking-based semantics σ satisfies *void precedence* iff for every $AF = \langle A, attacks \rangle$, for each $a, b \in A$ with $Att(a) = \emptyset$ and $Att(b) \neq \emptyset$, $a \succ^{\sigma}_{AF} b$ [21].

Self-Contradiction (SC) A ranking-based semantics σ satisfies *self-contradiction* iff for any $AF = \langle A, attacks \rangle$ with $a, b \in A$, $(a, a) \notin attacks$, and $(b, b) \in attacks$, $a \succ^{\sigma}_{AF} b$ [2].

Counter-Transitivity (CT) The postulates of *counter-transitivity* and *strict counter-transitivity* as defined in [2] rely on the concept of *group comparison*, i.e. comparing sets of arguments S_1, S_2 of an $AF = \langle A, attacks \rangle$ under a ranking-based semantics σ:

- $S_1 \succeq^{\sigma}_{AF} S_2$ iff $|S_1| \geq |S_2|$ and for any $s_2 \in S_2$ there is at least one $s_1 \in S_1$ with $s_1 \succeq^{\sigma}_{AF} s_2$.
- $S_1 \succ^{\sigma}_{AF} S_2$ iff $|S_2| > |S_1|$ or for any $s_2 \in S_2$ there is at least one $s_1 \in S_1$ with $s_1 \succ^{\sigma}_{AF} s_2$.

A ranking-based semantics σ satisfies *counter-transitivity* iff for any $AF = \langle A, attacks \rangle$, $\forall a, b \in A$ iff an argument a has a group of attackers at least as large and acceptable as the group of attackers of an argument b, then b should at least be ranked as high as a, i.e., iff $Att(a) \succeq^{\sigma}_{AF} Att(b)$ then $b \succeq^{\sigma}_{AF} a$ [2].

Strict Counter-Transitivity (SCT) A ranking-based semantics σ satisfies *strict counter-transitivity* iff for any $AF = \langle A, attacks \rangle$, $\forall a, b \in A$ iff the group of attackers of a is larger or has arguments more acceptable, then b should be ranked higher than a, i.e. iff $Att(a) \succ^{\sigma}_{AF} Att(b)$ then $b \succ^{\sigma}_{AF} a$ [2].

Cardinality Precedence (CP) A ranking-based semantics σ satisfies *cardinality precedence* iff for any $AF = \langle A, attacks \rangle$ with $a, b \in A$, and with $|Att(a)| > |Att(b)|$, $b \succ^{\sigma}_{AF} a$ [2].

Quality Precedence (QP) A ranking-based semantics σ satisfies *quality precedence* iff for any $AF = \langle A, attacks \rangle$ with $a, b \in A$ iff a has at least one attacker ranked higher than any attacker of b, then $b \succ^{\sigma}_{AF} a$.

Defense Precedence (DP) A ranking-based semantics σ satisfies *defense precedence* iff for any $AF = \langle A, attacks \rangle$, $\forall a, b \in A$ iff the number of attackers is the same for a, b ($|Att(a)| = |Att(b)|$), but b is only attacked by unattacked arguments, then $a \succ^{\sigma}_{AF} b$ [2].

Distributed-Defense Precedence (DDP) The defense of an argument a can be called *simple* iff every direct defender of a is attacking precisely one direct attacker of a.

The defense of an argument a can be called *distributed* iff every direct attacker of a is defended by, at most, one argument.

Given an $AF = \langle A, attacks \rangle$, with $a, b \in A$ having the same number of defenders and attackers, a ranking-based semantics σ satisfies *distributed defense precedence* iff for every a being protected by a simple and distributed defense and b only being protected by a simple defense, $a \succ^{\sigma}_{AF} b$ [2].

Global properties consider how defense and attack branches affect the ranking of an argument:

Attack vs. Full Defense (AvsFD) Given an $AF = \langle A, attacks \rangle$, with $a, b \in A$, *attack vs. full defense* is satisfied iff an argument a – without attack and only defense branches – and an argument b – that is attacked once by an unattacked argument – results in the ranking $a \succ^{\sigma}_{AF} b$ [2].

Addition of Defense Branch (+DB) Given an $AF = \langle A, attacks \rangle$, with $a, b \in A$ iff σ fulfills +DB, then the addition of a defense branch to any attacked argument a improves the ranking of a [21].

Strict Addition of Defense Branch (⨁DB) Given an $AF = \langle A, attacks \rangle$, with $a \in A$, iff σ fulfills $\bigoplus DB$, then the addition of a defense branch to any argument a improves the ranking of a [21].

Addition of Attack Branch (+AB) Given an $AF = \langle A, attacks \rangle$, with $a \in A$, iff σ fulfills $+AB$, then the addition of an attack branch to any argument a deteriorates the ranking of a [21].

Increase of Defense Branch (↑DB) Given an $AF = \langle A, attacks \rangle$, with $a \in A$, iff σ fulfills ↑DB, then increasing the length of a defense branch of a deteriorates the ranking of a [21].

Increase of Attack Branch (↑AB) Given an $AF = \langle A, attacks \rangle$, with $a \in A$, iff σ fulfills ↑AB, then increasing the length of an attack branch of a improves the ranking of a [21].

As the principles mentioned above for *ranking-based semantics* were found to be lacking (i.e., by [21]), additional properties for comparison of gradual semantics were introduced in [3].

Counting (CN) *Counting* considers the quantity of non-rejected attackers: When *CN* is fulfilled by a semantics σ, the following is true for any $AF = \langle A, attacks \rangle$ with $a, b \in A$ and $Deg^{\sigma}_{AF}(a) > 0$: Iff there is at least one argument $x \in A \backslash Att(a)$ with $Deg^{\sigma}_{AF}(x) > 0$ and $Att(b) = Att(a) \cup \{x\}$, then $Deg^{\sigma}_{AF}(a) > Deg^{\sigma}_{AF}(b)$.

Reinforcement (RN) *Reinforcement* says that increasing the strength of an attacker should lead to a decrease in strength for the attacked argument. A semantics σ satisfies *RN* iff for any $AF = \langle A, attacks \rangle$, with $a, b, x, y \in A$ if

- $Deg^{\sigma}_{AF}(a) > 0$ or $Deg^{\sigma}_{AF}(b) > 0$,
- $Att(a) \setminus Att(b) = \{x\}$,
- $Att(b) \setminus Att(a) = \{y\}$, and
- $Deg^{\sigma}_{AF}(y) > Deg^{\sigma}_{AF}(x)$,

then $Deg^{\sigma}_{AF}(a) > Deg^{\sigma}_{AF}(b)$.

Table 3: Fulfillment of postulates for different ranking-based semantics: Coloured cells contain results from [21, 19, 34, 56, 24, 8, 3, 1, 64].

	Postulates															
Sem.	SC	CT	SCT	QP	DP	+AB	↑DB	↑AB	AvsFD	CN	VP	DDP	+DB	CP	⨁DB	RN
hCat	×	✓	✓	×	✓	✓	✓	✓	×	✓	✓	×	×	×	×	✓
Mbs	×	✓	×	✓	×	×	×	×	✓	×	✓	×	×	×	×	✓
Embs	×	✓	×	✓	×	×	×	×	✓	×	✓	×	×	×	×	✓
Tbs	×	✓	×	✓	×	×	×	×	✓	×	×	×	×	×	×	✓
ITS	×	✓	×	✓	×	×	×	×	✓	×	×	×	×	×	×	✓
Count	×	✓	✓	×	✓	✓	✓	✓	×	✓	✓	×	×	×	×	✓
M&T	✓	×	×	×	×	✓	×	×	✓	×	✓	×	×	×	×	×
nsa	✓	×	×	×	×	×	×	×	×	✓	×	×	×	×	×	×

On the fulfillment of properties *Basic general properties* i.e., *Abs*, *NaE*, and *Tot* are fulfilled by all known ranking-based semantics and therefore of no interest to the evaluation [21]. *In* is fulfilled by nearly all gradual semantics we mentioned so far. Only for *Count* with different values of α, *In* is not satisfied [56].

Local and *global properties* are more relevant when comparing existing semantics. However, some contradict each other [21, 34]: If *CP* is fulfilled by a given semantics, then *QP*, *AvsFD*, or *+DB* cannot be fulfilled as well. If *VP* is satisfied, ⨁*DB* cannot be satisfied by the same semantics. If *SC* is fulfilled, then *CT*, *CP*, and *SCT* cannot be fulfilled.

Some of the *global* or *local* properties imply others [21, 34]: *SCT* implies *VP* and *CT*. *CT* implies *NaE*. *CT* and *SCT* imply *DP* and ⨁*DB* implies *+DB*. *VP* and *QP* imply *AvsFD*.

What properties are fulfilled by the gradual semantics discussed in this thesis can be seen in Table 3. However, the uncolored cells in Table 3 have not been examined in existing research so far. Thus, for *EMbs*, *Tbs*, *ITS*, *Count*, and *nsa*, we have to determine if those principles are satisfied.

Theorem 1. The *counting* semantics fulfills *CN* and *RN*.

Proof. Pu et al. [58] have shown for the *counting* semantics that – given an $AF = \langle A, attacks\rangle$ with $a, b \in A$ – for any value of α, iff $Att(a) \subset Att(b)$, then $a \succ b$. Thus, iff there is at least one argument $x \in A \setminus Att(a)$ and $Att(b) = Att(a) \cup \{x\}$, then $Deg^{Count}_{AF}(a) > Deg^{Count}_{AF}(b)$. Therefore, the *counting* semantics fulfills *CN*. As the *counting* semantics fulfills *SCT* and *SCT* implies *RN* [15], the *counting* semantics fulfills *RN*. □

Theorem 2. The *nsa* semantics does not fulfill *CN*, *RN*, *VP*, *+AB*, *DP*, *↑DB*, *QP*, *AvsFD*, and *↑AB*.

Proof. As it has been proven that *SC* is fulfilled by the *nsa* semantics, *CT*, *CP* as well as *SCT* cannot be fulfilled.

We have also shown by counterexample that the *nsa* semantics does not fulfill *+AB*, *DP*, *VP*, *↑DB*, *QP*, *AvsFD*, and *↑AB* (see Section 7 in the appendix). □

Theorem 3. *Embs* fulfills the same set of properties as *Mbs*, and *ITS* fulfills the same set of properties as *Tbs*.

Proof. To prove that *Embs* as well as *ITS* fulfill the postulates listed in Table 3, we can use the findings of Amgoud and Beuselinck [6] regarding equivalence relations between existing ranking-based semantics. When comparing ranking-based semantics, Amgoud and Beuselinck [6] discuss different notions of equivalence:

Refinement If a semantics S_1 *refines* a semantics S_2, then $\succeq_{S_1} \subseteq \succeq_{S_2}$, meaning S_1 adheres to the strict comparisons of S_2.

Weak equivalence If a semantics S_1 is *weakly equivalent* to S_2, then it does not have strict rankings opposite to the rankings in S_1. If only one semantics refines the other, then the semantics are weakly equivalent.

Strong equivalence If a semantics S_1 is *strongly equivalent* to S_2, then its arguments do have the same ranking $\succeq_{S_1} = \succeq_{S_2}$. In case both semantics refine each other, they are strongly equivalent.

Amgoud and Beuselinck [6] find that the pair of *Mbs* and *Embs* as well as the pair of *Tbs* and *ITS* are strongly equivalent. As this means that they provide the same ranking $\succeq_{S_1} = \succeq_{S_2}$, *Embs* fulfills the same set of properties as *Mbs*, and *ITS* fulfills the same set of properties as *Tbs*. □

3 Related Studies: Combining Extension- and Ranking-Based Semantics

As shown in the last chapters, the approaches of ranking- and extension-based semantics differ fundamentally. Whereas extension-based semantics return sets of arguments that can be accepted together, ranking-based semantics focus on evaluating the relative strength of an argument by assigning values or defining a ranking order [23].

Each approach has advantages and disadvantages. With extension-based semantics, arguments are either accepted or rejected, but a detailed evaluation of an argument's strength is missing [50]. While extension-based semantics perform a binary evaluation of argument strength, ranking-based semantics allow for a more nuanced assessment [23]. Not all arguments have the same impact: An attack can weaken another argument instead of defeating it [59]. By depicting attack relations this way, ranking-based semantics allow for considering the number of attackers, so there is a difference if one or multiple arguments attack an argument.

In existing research, there have been several attempts to study the relationship between existing ranking-based and classical extension-based semantics. Some studies have also tried to combine those two semantic families to get the benefits of both approaches.

An important aspect of existing research has been the *compatibility between gradual or ranking-based and extension-based semantics*.

Ranking order vs. acceptance status Bonzon et al. [23] mention that existing ranking-based semantics only evaluate an argument's relative strength: An argument's acceptance status as defined in existing classical extension-based semantics [38] cannot always be derived from the respective ranking order of a ranking-based semantics.

Blümel and Thimm [20] also notice an incompatibility between most existing ranking- and classical extension-based semantics, stating that admissible arguments are not necessarily ranked higher than rejected ones. They find that for a ranking-based semantics to be compatible with classical semantics, it cannot satisfy *SCT*, *CT*, *CP*, or *QP*.

Semantic equivalence Given an argumentation framework $AF = \langle A, attacks \rangle$, Amgoud and Ben-Naim [3] transform existing extension-based semantics σ_{ext} into a ranking semantics by using a scale $T = \{1, \alpha, \beta, 0\}$ with $1 > \alpha > \beta > 0$ and the acceptance status of an argument $a \in A$. If a is skeptically accepted, then $Deg_{AF}^{\sigma_{ext}}(a) = 1$. If a is credulously accepted, then $Deg_{AF}^{\sigma_{ext}}(a) = \alpha$. If a is rejected and not attacked by any extension, then $Deg_{AF}^{\sigma_{ext}}(a) = \beta$, otherwise $Deg_{AF}^{\sigma_{ext}}(a) = 0$.

Based on this transformation of existing extension-based semantics into ranking-based semantics, Amgoud and Beuselinck [6] take a closer look at the equivalence between existing ranking- and classical extension-based semantics and come to a slightly more nuanced conclusion for flat graphs:

- *Mbs* and *EMbs* semantics are found to be weakly equivalent with *stable*, or *preferred* semantics, and are found to refine *grounded* semantics.
- The ranking-based semantics *ITS* and *Tbs* semantics are determined to be weakly equivalent with *stable*, and *preferred* semantics, and strongly equivalent with *grounded* semantics.
- In contrast, the *h-categorizer* proves to be incompatible with *grounded*, *stable*, and *preferred* semantics.

While studies have shown that existing classical extension-based and ranking-based semantics have slightly different notions of acceptability, there have been various attempts to *improve existing ranking-based approaches with ideas from classical extension-based semantics*.

Refining ranking-based semantics Bonzon et al. [23] have suggested three new ways of refining the ranking $\succeq_{AF}^{\sigma 1}$ given by a ranking-based semantics, with a ranking $\succeq_{AF}^{\sigma 2}$ derived from extension-based or labeling-based semantics.

The first approach by Bonzon et al. [23] changes the ranking $\succeq_{AF}^{\sigma 1}$ given by a ranking-based semantics by lexicographically refining it with a ranking $\succeq_{AF}^{\sigma 2}$ derived from the acceptance status of the arguments under *complete*, *preferred*, *grounded*, or *stable semantics*.

The second approach by Bonzon et al. [23] uses the *justification status*. As suggested by Wu et al. [66], ranking-based semantics $\succeq_{AF}^{\sigma 1}$ can be refined by a ranking $\succeq_{AF}^{\sigma 2, JS\sigma}$ according to the justification status from *labeling-based semantics*. While Wu et al. focused only on *complete semantics*, Bonzon et al. [23] extend this approach to *preferred*, *stable*, and *grounded semantics*. The ranking $\succeq_{AF}^{\sigma 2, JS\sigma}$ is obtained by considering the hierarchy of the justification status, in which $\{in\} \succ \{in, undec\} \succ \{undec\} \simeq \{in, out, undec\} \simeq \{in, out\} \succ \{out, undec\} \succ \{out\}$.

The third approach by Bonzon et al. uses both the acceptance and justification status of the argument for the refinement of *propagation semantics* – a semantics based on the *propagation principle* [22], giving non-attacked arguments a greater impact.

Ranking-based semantics based on serializability Blümel and Thimm [20] improve ranking-based semantics with the help of ideas from extension-based semantics. A new family of ranking semantics based on serializability is developed: An extension is created by determining a minimal initial non-empty set S of admissible arguments under an extension-based semantics σ and then progressively adding more arguments in a serialization sequence – a method proposed in [61]. The rank of an argument is derived from the length of its shortest serialization sequence, i.e., its serialization index ser_{AF}. Concerning the principles of ranking-based semantics, the new ranking semantics $\succeq_{ser}$ fulfills the principles *Abs*, *In*, *Tot*, *NaE* as well as *AvsFD*, and *directionality*.

There also have been several studies *improving existing extension-based semantics with ideas from ranking-based semantics*:

Ranking extensions by considering principles In *graded semantics* as suggested by Grossi and Modgil [47], extensions from classical semantics are graded by considering the respective level of *conflict-freeness* and *self-defense*. An argument's justification status under different *graded* variants of classical semantics can be used to determine an argument ranking.

Skiba et al. [60] create a new *extension-ranking semantics* which ranks extensions of a classical extension-based semantics based on the respective level of *completeness* or *admissiblity*. Like Grossi and Modgil, Skiba et al. suggest using this extension ranking to derive a ranking of the individual arguments of an argumentation framework.

Ranking extensions with ranking-based semantics There have also been suggestions to use existing ranking-based semantics to improve existing extension-based semantics.

Bonzon et al. [23] propose that existing ranking-based semantics could make it possible to compare and evaluate extensions obtained by extension-based semantics and list possible approaches:

Selecting the best extensions To improve an existing extension-based semantics σ_1, one could filter each of the received extensions $E \in \sigma_1(AF)$ by additionally using a ranking-based semantics σ_2 and taking the rank of the arguments r_{σ_2} for all $x \in E$ into account (i.e. the *rank multiset* $rv_{\sigma_2}(E) = (r_{\sigma_2}(x_1)...r_{\sigma_2}(x_n))$.

In order to determine the score of an extension E, an aggregation function $\oplus$ like *sum*, *max*, *min*, *leximax*, or *leximin* can be used.

Extensions $E_1, E_2 \in \sigma_1(AF)$ could also be compared pairwise, based on the number of arguments more acceptable. After comparing all possible pairs, extensions with the best score could be selected.

Removing Attacks In this approach by Bonzon et al. [23], the results of the ranking-based semantics are given more importance by completely removing attacks from weaker to stronger arguments in an AF. Sets of accepted arguments are then computed under the chosen, extension-based semantics. By altering the AF, the resulting extensions are often not *conflict-free* in the original AF. To remedy this problem, those extensions must be shrunk to conflict-free subsets.

So far, there have been mainly abstract suggestions for *creating new extension-based semantics based on existing ranking-based semantics*.

Yu et al. [67] suggest using a generic modular framework for creating extensions with the help of existing ranking-based semantics. The framework consists of three layers: In the first layer, a *selection function* is used to select subsets of arguments, e.g., all maximal conflict-free or admissible sets. In the second layer, a ranking on the set of arguments is determined using any ranking-based semantics. In the third layer, a *lifting operator*, i.e., an aggregation function like *leximax*, is used to determine the strongest sets by evaluating the ranking of individual arguments.

Amgoud [1] has mentioned that it would be possible to create new extension-based semantics using the argument strength values given by an existing ranking-based semantics to determine whether an argument is part of an extension – an idea which will be further discussed in the next chapter.

4 Creating New Extension-Based Semantics With Gradual Semantics

As shown in the previous chapter, several studies have created new semantics by combining ideas from extension-based and ranking-based or gradual semantics.

The creation of new extension-based semantics based on values from gradual semantics, however, has not been the focus of existing research – except for Yu et al.'s and Amgoud's very abstract suggestions [67].

4.1 Approach and Formal Definition

This thesis will create new extension semantics σ_{ext_grad} based on different gradual semantics τ. Given an $AF = \langle A, attacks \rangle$ with $a \in A$, the argument strength $Deg^{\tau}_{AF}(a)$ and a threshold value δ will be used to determine whether an argument is accepted in an extension $E \in \sigma_{ext\_grad}(AF)$.

There are different possibilities for this approach, which will be explored regarding the threshold δ, the gradual semantics τ, and additional conditions for acceptance.

On choosing a gradual semantics τ Given the overview in Table 3, gradual semantics with different properties will be explored for τ. Additionally, τ should have a fixed range to facilitate the determination of a threshold δ s.t. given an $AF = \langle A, attacks \rangle$ and $a \in A$, $Deg^{\tau}_{AF}(a) \in [\beta, 1]$, with $\beta \geq 0$ denoting the minimum strength value for τ.

Based on these criteria, the following gradual semantics τ have been selected: the *h-Categorizer*, the *No self-attack h-Categorizer*, the *Max-Based*, the *Euler Max-Based*, the *Trust-Based*, the *Counting* as well as the *Iterative Schema*, and the *Matt and Toni* semantics.

Which gradual semantics is most suitable for creating a new extension-based semantics σ_{ext_grad} will be determined. A principle-based evaluation will explore how the properties of τ will influence the properties of the newly created semantics σ_{ext_grad}. More general conclusions about the relationship between properties of gradual- and extension-based semantics will be drawn.

On defining a threshold δ Different possibilities to use a threshold δ and a gradual semantics τ will be evaluated: As Amgoud has suggested in [1], there are several possibilities for deriving the acceptability of an argument from gradual semantics. We consider the following options: Deriving the acceptance status from the argument's strength, deriving the acceptance status from the strength of the argument's attackers, and comparing the argument's strength with the strength of its attackers.

Given an $AF = \langle A, attacks \rangle$, an argument $a \in A$ can only be part of an extension $E \in \sigma_{ext\_grad}(AF)$ iff one of the following conditions regarding τ and a threshold δ is met:

Absolute argument strength With *absolute argument strength*, an absolute threshold δ_{arg} for argument strength is defined, above which an argument will be accepted (iff $a \in E$, then $Deg^{\tau}_{AF}(a) > \delta_{arg}$). This approach has already been suggested by [1].

Relative argument strength With *relative argument strength*, an argument is accepted if its strength is higher than that of any of its attackers ($\forall b \in A$ such that $b \in Att(a)$, iff $a \in E$, then $Deg^{\tau}_{AF}(a) > Deg^{\tau}_{AF}(b)$). This approach has also already been proposed by [1].

Absolute attack strength With *absolute attack strength*, a threshold δ_{att} is defined for the strength of an argument's attackers such that for every $b \in A$ with $b \in Att(a)$ iff $a \in E$, then $Deg^{\tau}_{AF}(b) < \delta_{att}$.

On defining additional acceptance criteria Beyond argument strength, other factors could also be considered to determine whether an argument is accepted under σ_{ext_grad}. Given an AF and $E \in \sigma_{ext\_grad}(AF)$, the following acceptance criteria for E will be explored:

No additional acceptance criteria Without additional criteria, all accepted arguments (based on δ and τ) are in E.

Checking for admissibility If no stable threshold δ can be found for a gradual semantics τ without additional acceptance criteria, another way of finding an admissible extension would be to successively add the strongest arguments according to τ while still staying admissible – a solution which has already been suggested by [4].

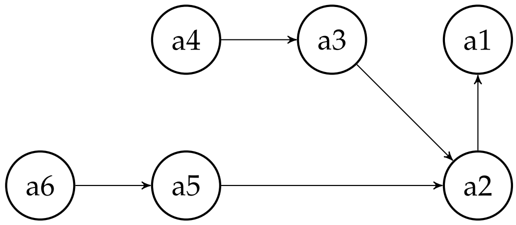

Figure 3: Abstract argumentation framework AF3

Formal definition σ_{ext_grad} consists of different extension-based semantics for which – given an $AF = \langle A, attacks \rangle$ and $a \in A$ – the values given by a gradual semantics τ with $Deg^{\tau}_{AF}(a) \in [\beta, 1]$ are used to determine if a is part of an extension $E \in \sigma_{ext\_grad}(AF)$.

Based upon the considerations in this chapter, we create and explore the following extension semantics σ_{ext_grad} based on a gradual semantics τ:

Ar- τ For the extension-based semantics $Ar\text{-}\tau$ and an $AF = \langle A, attacks\rangle$, an argument $a \in A$ is accepted iff $Deg^{\tau}_{AF}(a) > \delta_{arg}$ with $\delta_{arg} \in [\beta, 1)$. For every AF, there is only one extension $E \in Ar\text{-}\tau(AF)$. Iff for every argument $a \in A$, $Deg^{\tau}_{AF}(a) \leq \delta_{arg}$, then $E = \emptyset$.

Example 16. Given the $AF3$ from Figure 3, the argument values given by the *Mbs* semantics with $\epsilon = 0.0001$ are

- $Deg^{Mbs}_{AF}(a3) = Deg^{Mbs}_{AF}(a5) = 0.5$,
- $Deg^{Mbs}_{AF}(a1) = 0.6$,
- $Deg^{Mbs}_{AF}(a2) \approx 0.67$, and
- $Deg^{Mbs}_{AF}(a6) = Deg^{Mbs}_{AF}(a4) = 1$.

If we define $\delta_{arg} = 0.68$ for *Ar-Mbs*, $E \in$*Ar-Mbs*(AF) consists of $\{a6, a4\}$. If we define $\delta_{arg} = 0.65$ for *Ar-Mbs*, $E \in$*Ar-Mbs*(AF) consists of $\{a2, a6, a4\}$.

At- τ For the extension-based semantics $At\text{-}\tau$ and an $AF = \langle A, attacks\rangle$ with $a, b \in A$, an argument a is accepted iff for every $b \in Att(a)$ $Deg^{\tau}_{AF}(b) < \delta_{att}$ with $\delta_{att} \in (\beta, 1]$. For every AF, there is only one extension $E \in At\text{-}\tau(AF)$. Iff for every argument $a \in A$ with $b \in Att(a)$ $Deg^{\tau}_{AF}(b) \geq \delta_{att}$, then $E = \emptyset$.

Example 17. Given the $AF3$ from Figure 3 and the argument values computed for *Mbs* in the previous example, if we define $\delta_{att} = 0.6$ for *Ar-Mbs*, $E \in$*Ar-Mbs*(AF) consists of $\{a6, a2, a4\}$. If we define $\delta_{att} = 0.4$ for *Ar-Mbs*, $E \in$*Ar-Mbs*(AF) consists of $\{a6, a4\}$.

Re- τ For the extension-based semantics $Re\text{-}\tau$ and an $AF = \langle A, attacks\rangle$ with $a, b \in A$, an argument a is accepted iff for every $b \in Att(a)$ $Deg^{\tau}_{AF}(a) > Deg^{\tau}_{AF}(b)$. For every AF, there is only one $E \in Re\text{-}\tau(AF)$. Iff for every argument $a \in A$ with $b \in Att(a)$, $Deg^{\tau}_{AF}(a) \leq Deg^{\tau}_{AF}(b)$, then $E = \emptyset$.

Example 18. Given the $AF3$ from Figure 3 and the argument values computed for *Mbs*, $E \in$*Re-Mbs*(AF) would also consist of $\{a6, a2, a4\}$.

Ar- $\tau^{\mathbf{ad}}$ For the extension-based semantics $Ar\text{-}\tau^{ad}$ and an $AF = \langle A, attacks\rangle$, an argument $a \in A$ is accepted in $E \in Ar\text{-}\tau^{ad}(AF)$ with $E \subseteq A$ iff $Deg^{\tau}_{AF}(a) > \delta^{AF}_{ad}$. The threshold δ^{AF}_{ad} is determined for each AF s.t. $E \in adm(AF)$, and E is maximal s.t. there is no threshold $\delta 2^{AF}_{ad} < \delta^{AF}_{ad}$ for which $E_2 \subseteq A$ with $E_2 \in Ar\text{-}\tau^{ad}(AF)$ is admissible and $|E_2| > |E|$. For every AF, there is only one $E \in Ar\text{-}\tau^{ad}(AF)$. Iff for every argument $a \in A$, $Deg^{\tau}_{AF}(a) \leq \delta^{AF}_{ad}$, then $E = \emptyset$.

Example 19. Given the $AF3$ from Figure 3 and the argument values computed for *Mbs*, $E \in$*Ar-Mbs*$^{\mathbf{ad}}(AF)$ would also consist of $\{a6, a2, a4\}$ with $\delta^{AF}_{ad} < 0.67$.

4.2 Implementation

After formally defining our new semantics, we will now discuss the implementation. The new extension-based semantics $Ar\text{-}\tau$,$At\text{-}\tau$, $Re\text{-}\tau$ as well as $Ar\text{-}\tau^{ad}$ were implemented using the *Tweety Project*[7], which was extended with a new project.[8]

To be able to deal with cycles in an $AF = \langle A, attacks \rangle$, all gradual semantics except for those based on *M&T* had to be implemented using the fixed point iteration technique suggested by Pu et al. [57]. With this technique, Deg^{τ}_{AF} for an argument $a \in A$ is computed iteratively for k steps until the change to the approximate solution v^k is under a given tolerance ϵ s.t. $\|v^k - v^{k-1}\| < \epsilon$. For the *counting* semantics, we use $\alpha = 0.9$ for all evaluations, as this value has been recommended by Pu et al. in [56].

To test the *suitability* of a gradual semantics τ, experimental evaluations were performed for all new semantics. All the newly implemented semantics were experimentally evaluated for the following principles: *I-maximality*, *weak reinstatement*, *CF-reinstatement*, *reduct admissibility*, *defense*, *directionality*, *SCC recursiveness*, *modularization*, *semi-qualified admissibility*, *reinstatement*, *conflict-freeness*, *INRA*, *admissibility*, *naivety* and *strong admissibility*.

We declared principles to be *potentially fulfilled* if they could not be disproven in the experimental evaluations.

As test data for the *experimental evaluations*, three different test sets were used:

- For most *experimental evaluations* in this chapter, 124 selected graphs from the *ICCMA 17* and *ICCMA 19*[9] competition, 1000 self-generated graphs (generated with the *IsoSafeEnumeratingDungTheoryGenerator* from the *Tweety Project*) and 31 selected graphs from [14], [13], [63] and [33] were used. Overall, the test set consisted of 1155 graphs. Among those, 46 graphs had no cycles at all. Among the 1109 graphs with cycles, 841 graphs included self-attacking arguments, and 980 had odd cycles. The number of SCCs in the argumentation frameworks ranged from 1 to 90. The number of arguments per graph ranged from 1 to 103, and the number of attacks from 0 to 5094.

- Due to its high computational complexity [19], a reduced test set with smaller argument graphs had to be used for the *M&T* semantics to prevent out-of-memory errors. This reduced test set consisted of 500 self-generated graphs (generated with the *IsoSafeEnumeratingDungTheoryGenerator* from the *Tweety Project*). Among those, 4 graphs had no cycles at all. Among the 496 graphs with cycles, 411 graphs included self-attacking arguments, and 461 had odd cycles. The number of $SCCs$ in the argumentation frameworks ranged from 0 to 4.

[7] `https://github.com/TweetyProjectTeam/TweetyProject`

[8] `https://github.com/carolakatharina/TweetyProject/`

[9] The benchmark graphs were taken from `http://argumentationcompetition.org/2017/` and `http://argumentationcompetition.org/2019/`.

- To test the *stability* of the newly created semantics, a third test set containing selected, self-constructed edge-case argumentation frameworks has been used.

4.2.1 Implementing *Ar*-τ and *At*-τ

For the implementation of *Ar*-τ, Algorithm 1 has been used. For the implementation of *At*-τ, Algorithm 2 has been applied. As stated, both *Ar*-τ and *At*-τ provide only one extension per argumentation framework. If the acceptance condition is not met for any argument, then $E = \emptyset$.

Algorithm 1 Determining the *Ar*-τ-extension

Input: a directed graph $AF = \langle A, attacks \rangle$,
a gradual semantics τ,
a threshold $\delta_{arg} \in [\beta, 1)$
Output: $E \subseteq A$ with $E \in$ *Ar*-τ(AF)

$ranking \leftarrow$ empty map
for each a in A **do**
  $key \leftarrow a$
  $value \leftarrow Deg^{\tau}_{AF}(a)$
  add $(key, value)$ to $ranking$
end for each
$E \leftarrow$ empty extension
for each $entry$ in $ranking$ **do**
  if ($entry.value$>δ_{arg}) **then**
    add $entry.key$ to E
  end if
end for each
return E

Experimental evaluation To determine whether a gradual semantics τ can be used for *Ar*-τ resp. *At*-τ, and which threshold δ_{att} resp. δ_{arg} should be selected for each τ, an experimental evaluation was conducted.

For each of the *Ar*-τ resp. *At*-τ implemented, different values for δ_{att} resp. δ_{arg} were tested against the previously defined test data.

The experimental evaluation was conducted in several steps for each gradual semantics τ:

1. To evaluate the behavior *Ar*-τ resp. *At*-τ different values of δ_{arg} resp. δ_{att} were explored: Starting with the lowest possible value, 0.001 was added to that value until the maximum value was reached.
2. For those gradual semantics using the fixed point iteration technique suggested by Pu et al. [57], it was important to determine whether the value for ϵ influenced the behavior of *Ar*-τ resp. *At*-τ: Thus, different values for ϵ with $\epsilon \in \{0.01, 0.001, 0.0001, 0.00001\}$ were explored in a more detailed threshold evaluation with distances of 0.0001. *M&T* is the only gradual semantics for which the fixed point technique was not used, so there is no detailed evaluation for different values of ϵ.

Algorithm 2 Determining the *At*-τ-extension

Input: a directed graph $AF = \langle A, attacks \rangle$,
a gradual semantics τ,
a threshold $\delta_{att} \in (\beta, 1]$
Output: $E \subseteq A$ with $E \in$ *At*-τ(AF)

$ranking \leftarrow$ empty map
for each a in A **do**
  $key \leftarrow a$
  $value \leftarrow Deg^{\tau}_{AF}(a)$
  add $(key, value)$ to $ranking$
end for each
$E \leftarrow$ empty extension
for each a in A **do**
  $attackers \leftarrow Att(a)$
  $inExt \leftarrow true$
  for each att in $attackers$ **do**
    $attEntry \leftarrow ranking.get(att)$
    if $(attEntry.value \geq \delta_{att})$ **then**
      $inExt \leftarrow false$
    end if
  end for each
  if $inExt$ **then**
    add $entry.key$ to E
  end if
end for each
return E

3. A potentially *optimal* threshold δ_{arg} resp. δ_{att} was determined for *Ar*-τ resp. *At*-τ. We considered a threshold δ_{arg} resp. δ_{att} to be potentially *optimal* if the number of potentially fulfilled extension-based principles was maximal and *admissibility* was potentially fulfilled.

   If no potentially optimal $\delta_{arg} < 1$ resp. $\delta_{att} > \beta$ (with β denoting the minimum strength value for τ) could be found, the gradual semantics τ was considered to be potentially *unsuitable* for creating *Ar*-τ resp. *At*-τ.

4. For all *Ar*-τ resp. *At*-τ with a potentially optimal threshold δ_{arg} resp. δ_{att} for creating *Ar*-τ resp. *At*-τ, the *stability* of the thresholds was explored. For this, selected, self-constructed edge-case argumentation graphs were used.

   We declared the potentially optimal threshold δ_{arg} resp. δ_{att} to be *unstable* if *admissibility* could be experimentally disproven for *Ar*-τ resp. *At*-τ using other argumentation frameworks as test data. If a threshold δ_{arg} resp. δ_{att} proved to be *unstable*, the gradual semantics τ was also declared potentially unsuitable for the creation of *Ar*-τ resp. *At*-τ.

Given an argumentation framework $AF = \langle A, attacks \rangle$ and $a \in A$, for *Mbs*, $Deg^{Mbs}_{AF}(a)$ lies in the interval $[\frac{1}{2}, 1]$ [5]. For $\tau \in \{$*Embs*, *ITS*, *M&T*, *Count*,*Tbs*$\}$, $Deg^{\tau}_{AF}(a)$ lies in the range $[0, 1]$ [7, 46, 32, 52, 56, 19]. For the *hCat* semantics, $Deg^{hCat}_{AF}(a)$ can assume values in $(0, 1]$ [18]. Thus, the subsequent threshold evaluations have been executed using these ranges.

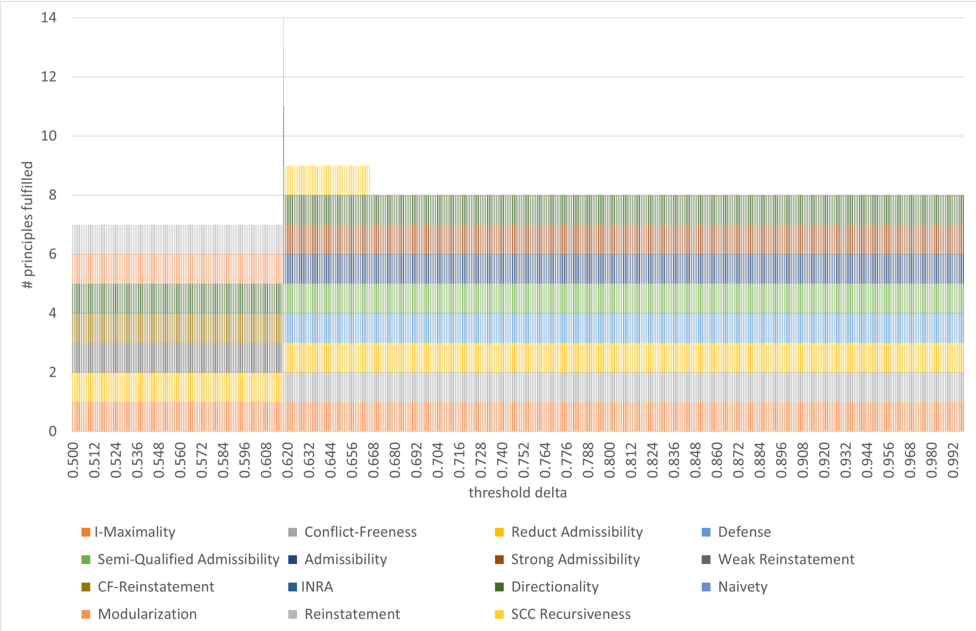

Figure 4: Principles fulfilled for *Ar-Mbs* with $\epsilon = 0.0001$ for different values of δ_{arg}. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

Determining δ_{arg} for Ar-τ When using *absolute argument strength* in an experimental threshold evaluation, the gradual semantics $\tau \in \{Mbs, Embs, ITS, Tbs, M\&T\}$ were found to be potentially suitable for the creation of *Ar*-τ:

Ar-Mbs and Ar-Embs For *Ar-Mbs* with $\epsilon = 0.0001$, threshold values of $0.618 < \delta_{arg} < 0.6182$ returned the best results: Only *naivety* could be experimentally disproven (see Figure 4). For *Ar-EMbs* with $\epsilon = 0.0001$, the threshold values $0.5671 < \delta_{arg} < 0.579$ proved to be most promising: All principles except for *naivety* were potentially fulfilled (see Figure 5).

For *Ar-Mbs* with $\delta_{arg} < 0.6181$ and *Ar-Embs* with $\delta_{arg} < 0.5672$, *conflict-freeness*, *INRA*, *admissibility*, *SCC recursiveness* and *strong admissibility* could be experimentally disproven. For *Ar-Mbs* with $\delta_{arg} > 0.619$ and *Ar-Embs* with $\delta_{arg} > 0.579$, *weak reinstatement*, *CF-reinstatement*, *reinstatement*, *modularization* and *INRA* could be experimentally shown to be not satisfied.

Using different values for ϵ, the behavior varied slightly for *Ar-Mbs* and *Ar-Embs* (see Figure 32, 31, 34, and 35 in the appendix).

Based on these experimental results, the following potentially *optimal* thresholds for $\epsilon = 0.0001$ were selected: For *Ar-Mbs*, $\delta_{arg} = 0.6181$ was determined, for *Ar-Embs* $\delta_{arg} = 0.5672$ was selected. When re-running a threshold evaluation for *Ar-Mbs* resp. *Ar-Embs* against self-constructed edge-case argumenta-

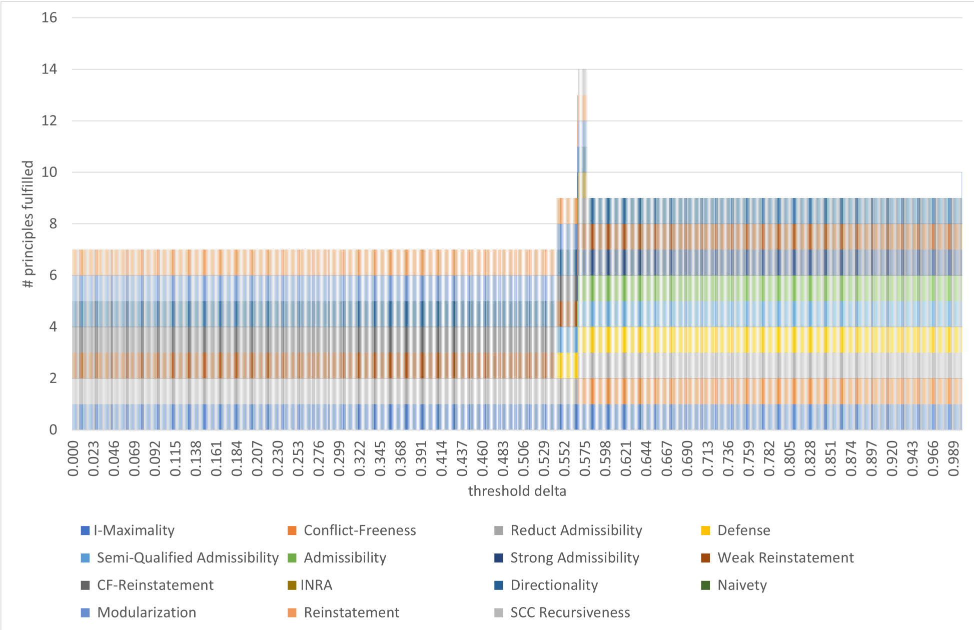

Figure 5: Principles fulfilled for *Ar-Embs* with $\epsilon = 0.0001$ for different values of δ_{arg}. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

tion frameworks, the thresholds were stable.

Ar-Tbs and Ar-ITS In [6], Amgoud and Beuselinck have shown that in the case of flat graphs, *Tbs* and *ITS* assign mostly the same values to arguments, i.e., they coincide. Consequently, both experimental threshold evaluations showed minimal differences. The maximum number of 14 potentially fulfilled principles for *Ar-Tbs* and *Ar-ITS* with $\epsilon = 0.0001$ was reached for $\delta_{arg} \geq 0.5$: For those threshold values, only *naivety* could be experimentally disproven (see Figure 6).

For $\delta_{arg} < 0.5$, *conflict-freeness*, *INRA*, *SCC recursiveness*, *admissibility* and *strong admissibility* were not fulfilled in the experimental evaluation.

Using different values for ϵ led to slight variances in the behavior of *Ar-ITS* and *Ar-Tbs* (see Figure 38 and 37 in the appendix).

Based on these experimental results, the potentially *optimal* threshold $\delta_{arg} = 0.5$ was selected for $\epsilon = 0.0001$. When re-running a threshold evaluation for *Ar-Tbs* resp. *Ar-ITS* against self-constructed edge-case argumentation frameworks, the thresholds were stable.

Ar-M&T For *Ar-M&T*, in the experimental threshold evaluation, the maximum number of potentially fulfilled principles was reached for $\delta_{arg} \geq 0.5$ (see Fig-

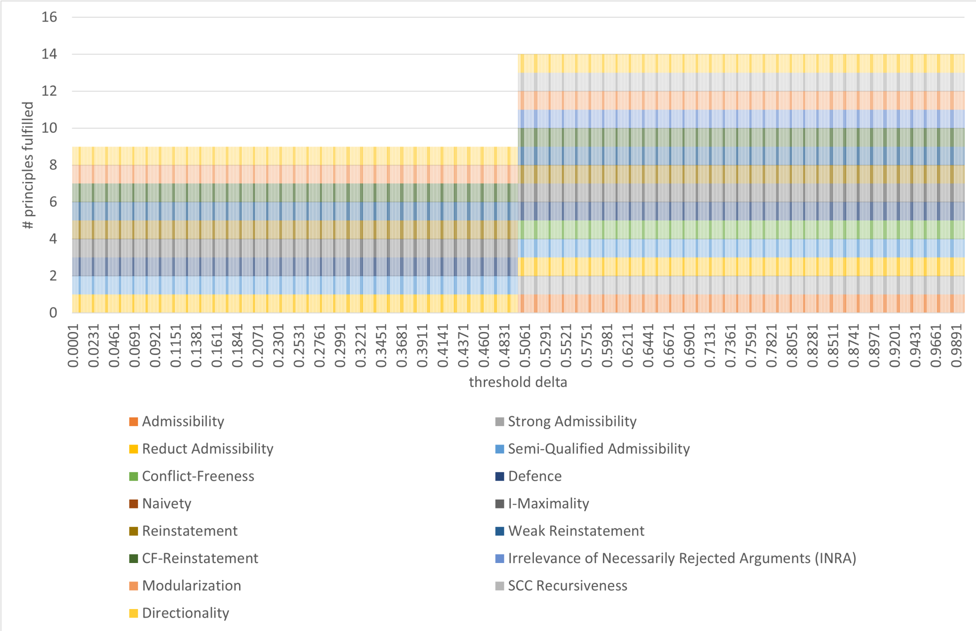

Figure 6: Principles fulfilled for *Ar-ITS* resp. *Ar-Tbs* with $\epsilon = 0.0001$ for different values of δ_{arg}. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

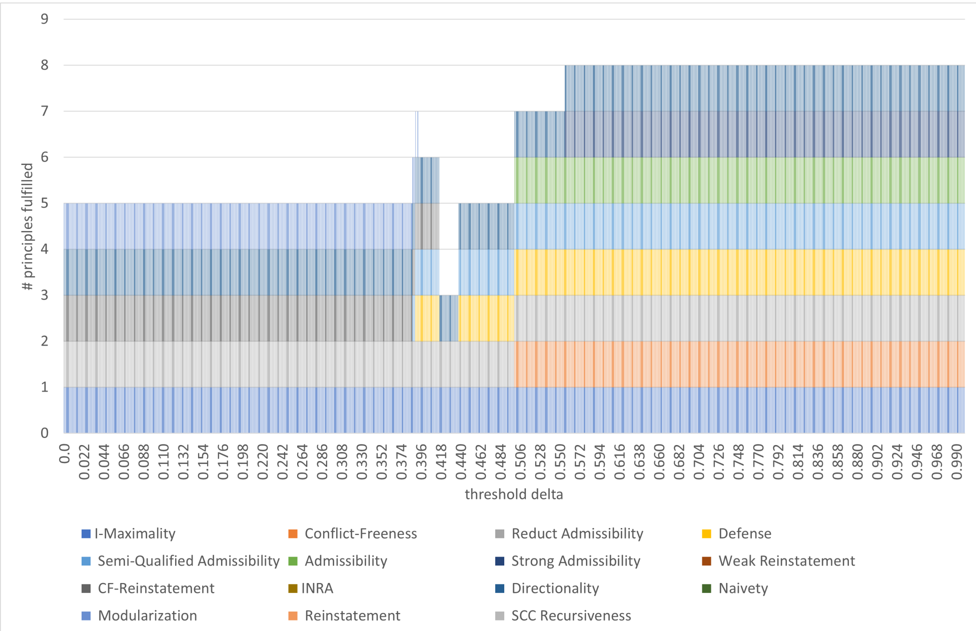

Figure 7: Principles fulfilled for *Ar-M&T* for different values of δ_{arg}. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

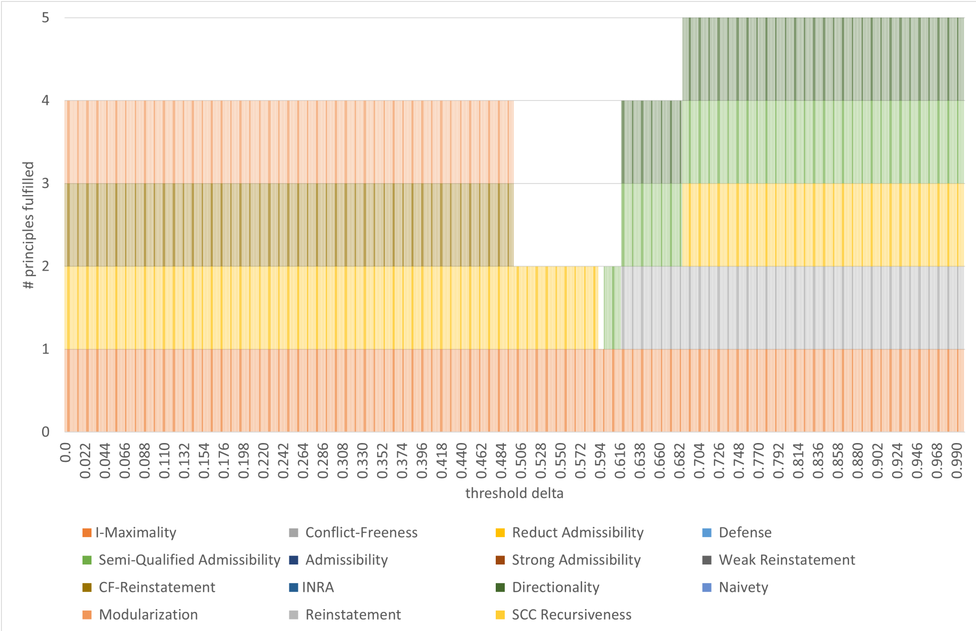

Figure 8: Principles fulfilled for *Ar-nsa* for different values of δ_{arg}. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

ure 7 and Figure 49 in the appendix). *INRA*, *naivety*, *SCC recursivess*, and *reinstatement* as well as *weak reinstatement* were experimentally disproven for any value of δ_{arg} explored. *Conflict-freeness* and *admissibility* were experimentally disproven for $\delta_{arg} < 0.5$. Based on these experimental results, the potentially *optimal* threshold of $\delta_{arg} = 0.5$ was determined for *Ar-M&T*. When re-running a threshold evaluation for *Ar-M&T* against self-constructed edge-case argumentation frameworks, this threshold was stable.

When using *absolute argument strength* in an experimental threshold evaluation, the gradual semantics $\tau \in \{hCat, nsa, Count\}$ were found to be unsuitable for the creation of *Ar-*τ:

Ar-nsa For *Ar-nsa* with $\epsilon = 0.0001$, at most 5 principles were potentially fulfilled for $\delta_{arg} \geq 0.687$ (see Figure 8). *Modularization* and *CF-reinstatement* could be experimentally disproven for $\delta_{arg} > 0.5$. *Reinstatement* as well as *weak reinstatement*, *admissibility*, *strong admissibility*, *naivety*, *INRA* and *SCC recursivess* could be experimentally disproven for all values of δ_{arg}. *Semi-qualified admissibility* was not fulfilled for $\delta_{arg} < 0.6$ in the experimental evaluation, *conflict-freeness* was not satisfied for $\delta_{arg} < 0.619$.

For different values of ϵ, the behavior of *Ar-nsa* did not vary (see Figure 44 and Figure 43 in the appendix).

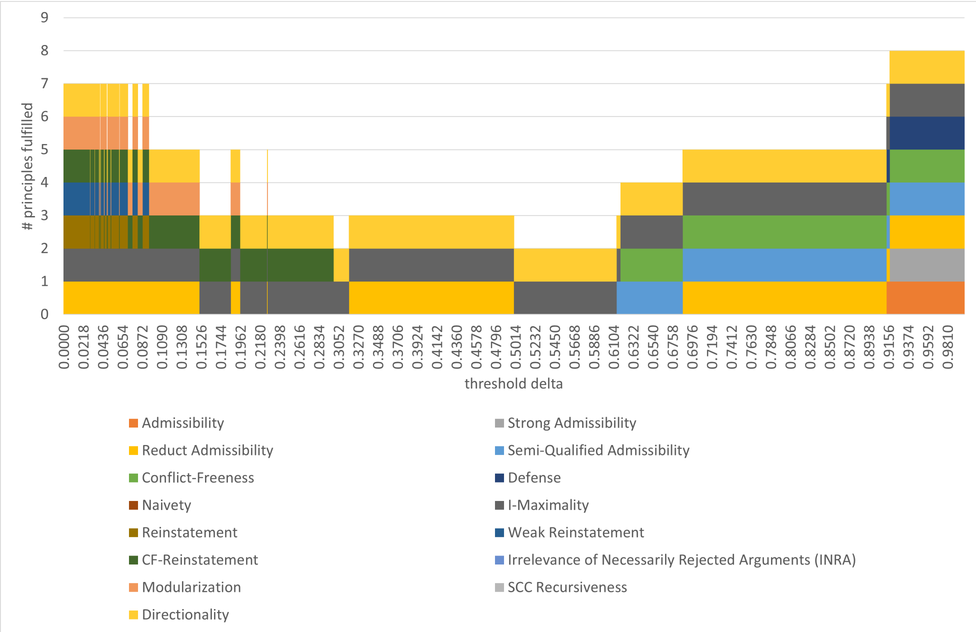

Figure 9: Principles fulfilled for *Ar-hCat* with $\epsilon = 0.0001$ for different values of δ_{arg}. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

As *admissibility* was not fulfilled for any value of δ_{arg} in our experimental evaluation, *nsa* was considered to be potentially *unsuitable* for the creation of *Ar-*τ.

Ar-hCat For *Ar-hCat* with $\epsilon = 0.0001$, at most 8 principles were potentially fulfilled (see Figure 9). For $\delta_{arg} = 0.917$, only *modularization*, *reinstatement*, *weak reinstatement*, *CF-reinstatement*, *naivety*, *INRA*, and *SCC recursiveness* could be experimentally disproven.

Semi-qualified admissibility could be experimentally disproven for $\delta_{arg} \leq 0.613$, *Conflict-freeness* for $\delta_{arg} < 0.619$, *admissibility* for $\delta_{arg} \leq 0.913$ and *strong admissibility* for $\delta_{arg} \leq 0.916$.

Modularization was not fulfilled for $\delta_{arg} \geq 0.197$ in the experimental evaluation; *reinstatement* as well as *weak reinstatement* were experimentally disproven for $\delta_{arg} \geq 0.135$ and *CF-reinstatement* for $\delta_{arg} \geq 0.299$. *Naivety*, *INRA* or *SCC recursivess* were not fulfilled for any of the values used for δ_{arg} in the experimental evaluation.

For different values of ϵ, the behavior for *Ar-hCat* did not vary significantly (see Figure 41 and Figure 40 in the appendix).

Whereas the results for *Ar-hCat* were promising at first glance, the potentially optimal threshold $\delta_{arg} = 0.917$ was unstable: When re-running a threshold

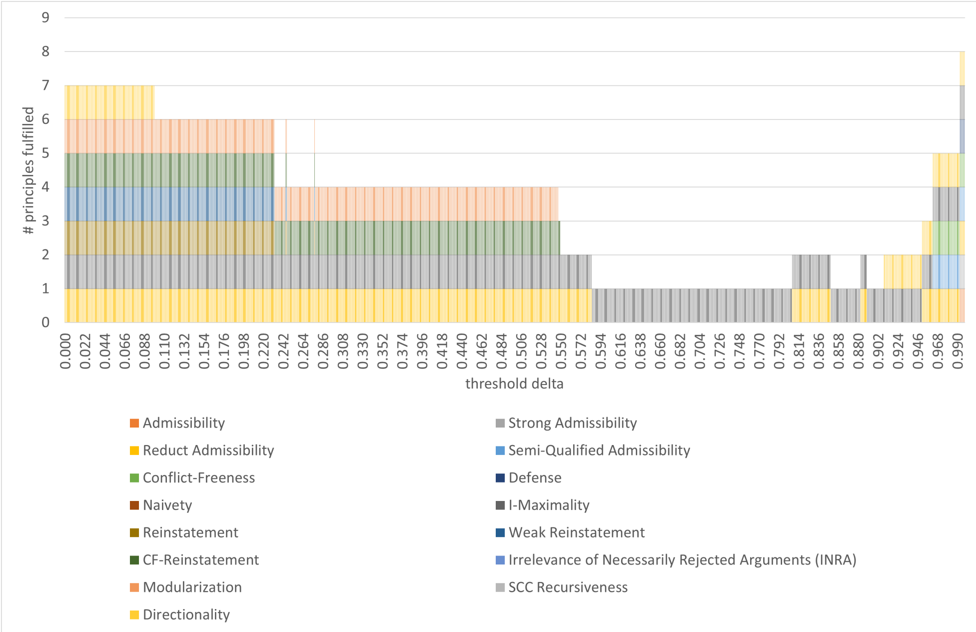

Figure 10: Principles fulfilled for *Ar-Count* for different values of δ_{arg}. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

evaluation for *Ar-hCat* against selected edge-case argumentation frameworks, *admissibility* could be experimentally disproven when using $\delta_{arg} = 0.917$.

Ar-Count The maximum number of potentially fulfilled for the *Ar-Count* semantics was 8 and was reached for $\delta_{arg} \geq 0.994$ for $\epsilon = 0.0001$ (see Figure 10).

For $\delta_{arg} > 0.278$, *weak reinstatement*, and *reinstatement* could be experimentally disproven, for $\delta_{arg} > 0.550$ *CF-reinstatement* was not fulfilled. *Conflict-freeness* and *semi-qualified admissibility* could not be experimentally disproven for $\delta_{arg} \geq 0.964$. *Admissibility* and *strong admisssibility* as well as *defense* were potentially fulfilled for $\delta_{arg} \geq 0.994$. *Naivety*, *SCC recursiveness*, and *INRA* could be disproven for all values of δ_{arg} used in the experimental evaluation.

An evaluation for different values of ϵ revealed that the behavior of *Ar-Count* did not vary significantly (see Figure 47 and Figure 46 in the appendix).

Just like for *Ar-hCat*, whereas the results for *Ar-Count* were promising at first glance, the potentially optimal threshold δ_{arg} proved to be unstable. When re-running a threshold evaluation for *Ar-Count* with $\delta_{arg} = 0.994$ against selected edge-case argumentation frameworks, *admissibility* could be experimentally disproven as well. Thus, *Ar-Count* was also declared potentially unsuitable for creating *Ar-*τ.

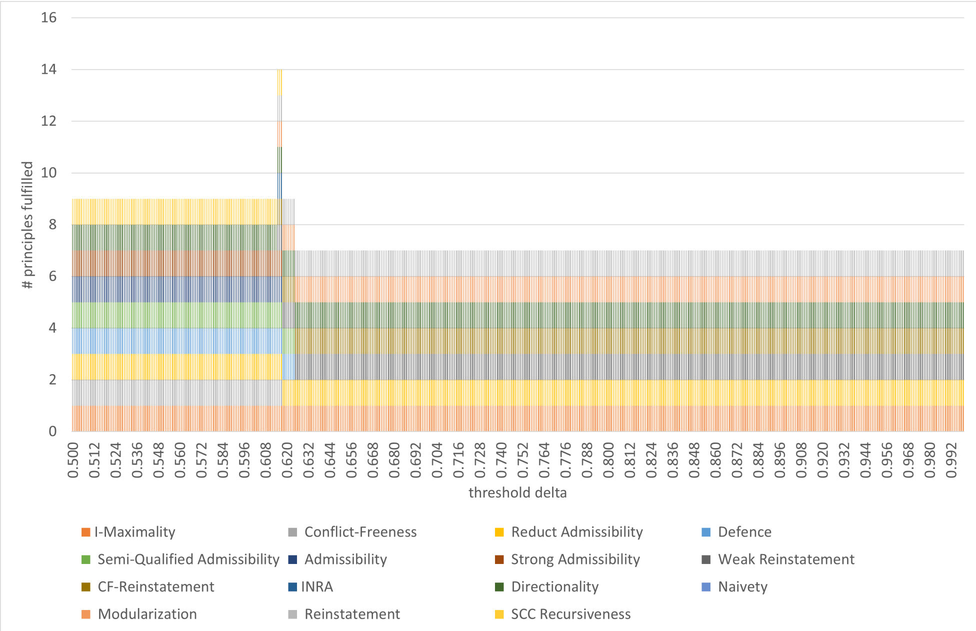

Figure 11: Principles fulfilled for *At-Mbs* with $\epsilon = 0.0001$ for different values of δ_{att}. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

Determining δ_{att} for At-τ When using *absolute attack strength* in an experimental threshold evaluation, only the gradual semantics $\tau \in \{Mbs, Embs, ITS, Tbs\}$ were found to be potentially suitable for the creation of *At-*τ:

At-Mbs For *At-Mbs* with $\epsilon = 0.0001$, the highest number of potentially fulfilled principles was reached for $0.616 \leq \delta_{att} \leq 0.618$: Here, only *naivety* could be experimentally disproven (see Figure 11).

For $\delta_{att} < 0.616$, *weak reinstatement*, *CF-reinstatement*, *reinstatement*, *modularization*, and *INRA* could be experimentally disproven. For $\delta_{att} > 0.618$, *conflict-freeness*, *INRA*, *SCC recursiveness*, *admissibility* and *strong admissibility* were not fulfilled. *Naivety* was experimentally disproven for any value of δ_{att} explored.

For different values of ϵ, the behavior of *At-Mbs* did not vary significantly (see Figure 33 in the appendix).

Regarding *At-Mbs* with $\epsilon = 0.0001$, the value 0.618 was evaluated as a potentially optimal threshold δ_{att}. When re-running a threshold evaluation for *At-Mbs* against selected edge-case argumentation frameworks, the threshold $\delta_{att} = 0.618$ remained stable.

At-Embs For *At-Embs* with $\epsilon = 0.0001$, the best results in the experimental threshold evaluation were returned for $0.546 \leq \delta_{att} \leq 0.567$: Only *naivety* could be experimentally disproven in this range. For $\delta_{att} < 0.546$, *weak reinstatement*,

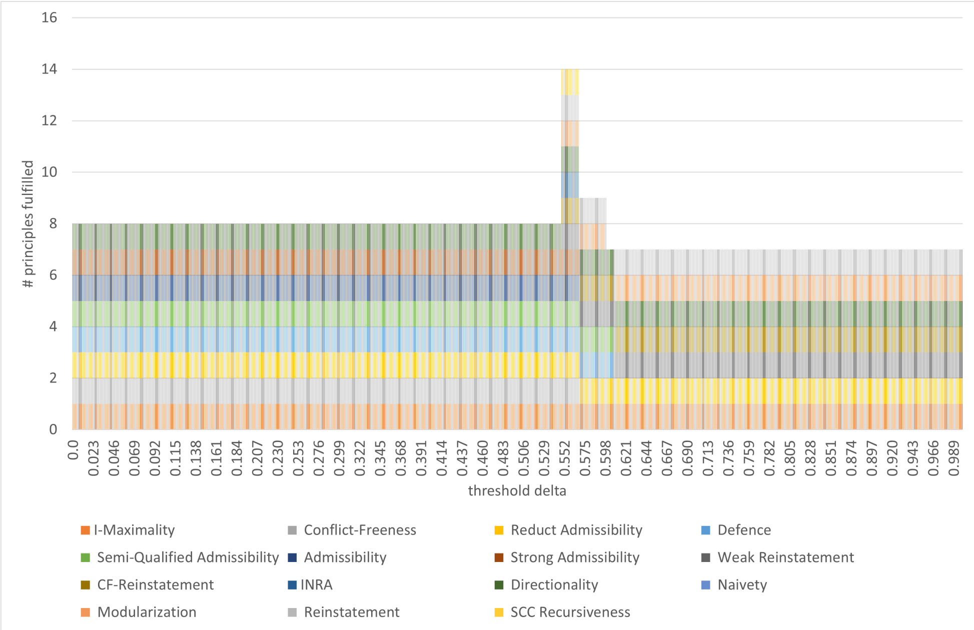

Figure 12: Principles fulfilled for *At-Embs* with $\epsilon = 0.0001$ for different values of δ_{att}. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

CF-reinstatement, and *reinstatement*, and *modularization* were not fulfilled in the evaluation. For $\delta_{att} > 0.567$, *conflict-freeness*, *INRA*, *SCC recursiveness*, *admissibility* and *strong admissibility* could be experimentally disproven. *Naivety* was never fulfilled for any value of δ_{att} used.

For different values of ϵ, the values for the potentially optimal δ_{att} did not vary significantly (see Figure 36 in the appendix).

Based on these experimental results, the potentially optimal *absolute attack strength* threshold for *At-Embs* was determined to be $\delta_{att} = 0.567$ for $\epsilon = 0.0001$. Concerning the stability of the threshold for *At-Embs*, $\delta_{att} = 0.567$ returned the same results in our tests against edge-case argumentation frameworks.

At-ITS and At-Tbs For *At-ITS* resp. *At-Tbs* with $\epsilon = 0.0001$, the highest number of potentially fulfilled principles was reached for $\delta_{att} < 0.5$: For these values of δ_{att}, only *naivety* could be experimentally disproven (see Figure 13). For $\delta_{att} \geq 0.5$, *conflict-freeness*, *INRA*, *SCC recursiveness*, *admissibility* and *strong admissibility* were not fulfilled in the experimental evaluation.

For different values of ϵ, the behavior of *At-ITS* resp. *At-Tbs* did not vary significantly (see Figure 39).

Based on our results, we determined $\delta_{att} = 0.49999$ as a potentially optimal

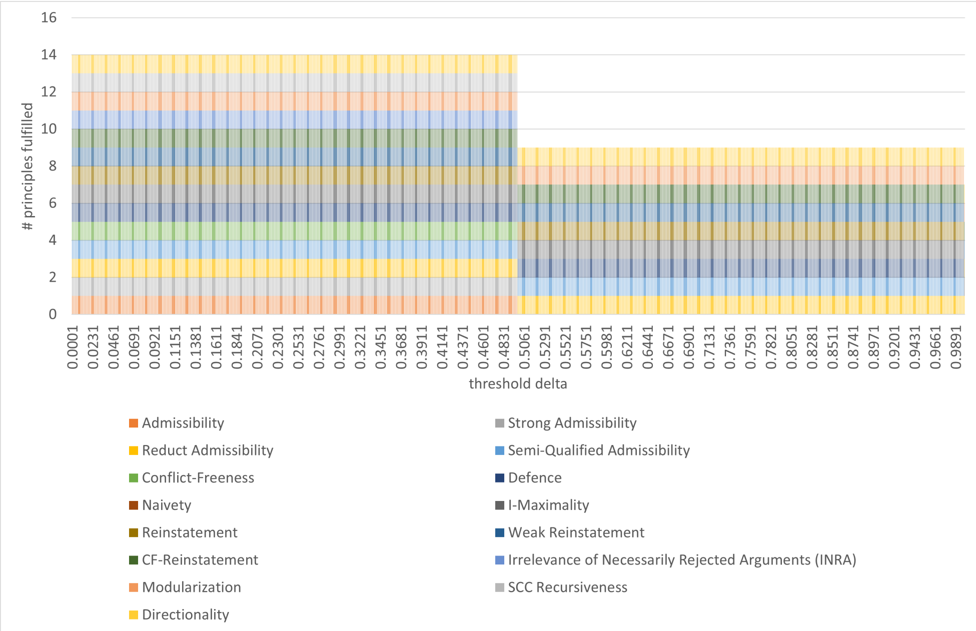

Figure 13: Principles fulfilled for *At-ITS* resp. *At-Tbs* with $\epsilon = 0.0001$ for different values of δ_{att}. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

absolute attack strength threshold for *At-ITS* resp. *At-Tbs* with $\epsilon = 0.0001$. The threshold was stable when tested against edge case frameworks.

When using *absolute attack strength* in an experimental threshold evaluation, the gradual semantics $\tau \in \{hCat, nsa, M\&T, Count\}$ proved to be potentially unsuitable for the creation of $At\text{-}\tau$:

At-M&T For *At-M&T*, the experimental evaluation showed that at most, 8 principles could be potentially fulfilled for $\delta_{att} \geq 0.251$ (see Figure 14 and Figure 50 in the appendix).

For $\delta_{att} < 0.251$, *weak reinstatement*, *CF-reinstatement*, *modularization*, *reduct admissibility*, and *reinstatement* could be experimentally disproven. *Naivety*, *SCC recursiveness*, *INRA*, *conflict-freeness*, *admissibility*, *defense* and *strong admissibility* were never fulfilled for *At-M&T* for any values of δ_{att} in the experimental evaluation.

As *admissibility* and *conflict-freeness* could be experimentally disproven for all values of δ_{att} used, *M&T* was deemed to be potentially *unsuitable* for the creation of *At-τ*.

At-Count For the *At-Count* semantics, the maximum number of principles potentially fulfilled was reached for $\delta_{att} \leq 0.229$ for $\epsilon = 0.0001$ (see Figure 15).

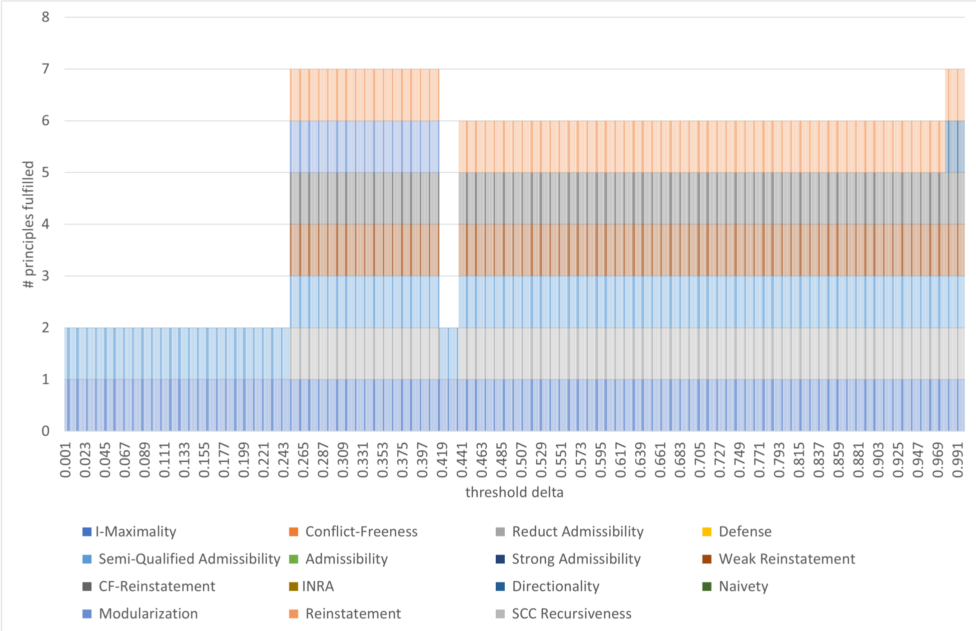

Figure 14: Principles fulfilled for *At-M&T* for different values of δ_{att}. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

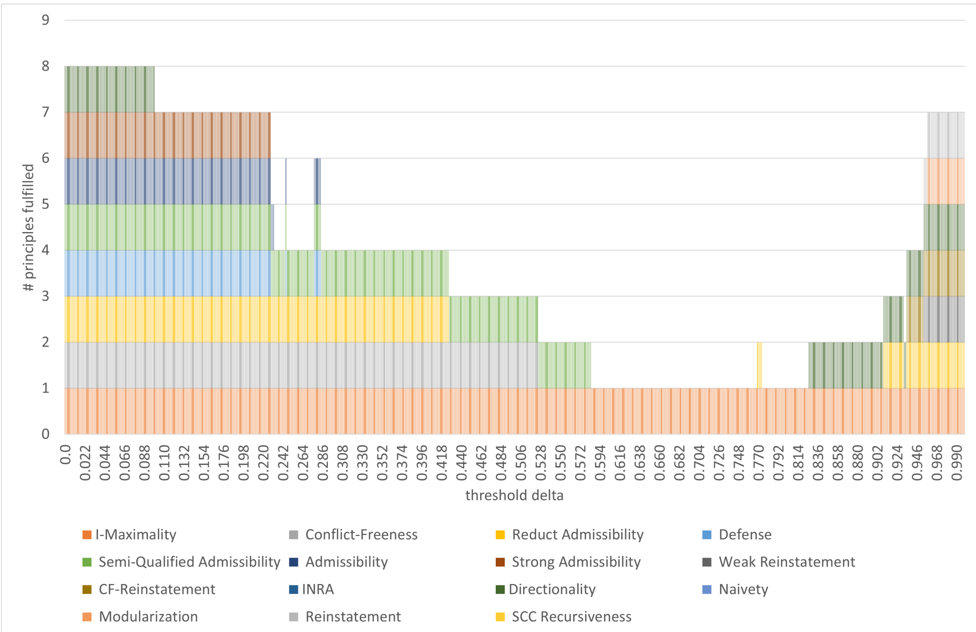

Figure 15: Principles fulfilled for *At-Count* for different values of δ_{att}. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

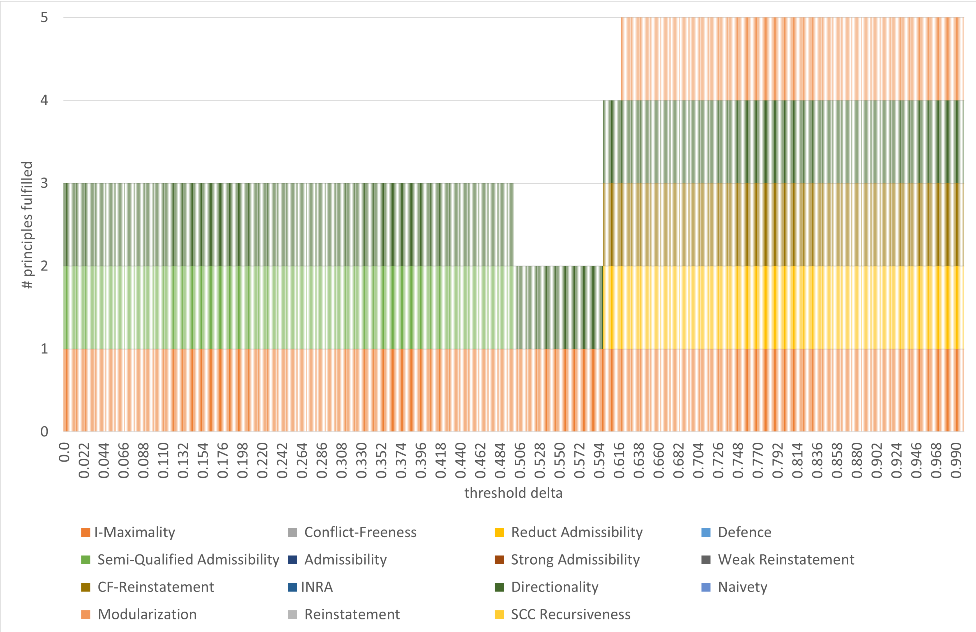

Figure 16: Principles fulfilled for *At-nsa* with $\epsilon = 0.0001$ for different values of δ_{att}. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

For $\delta_{att} > 0.229$, *strong admissibility* was experimentally disproven, for $\delta_{att} > 0.285$ *admissibility* was not consistently satisfied in the experimental evaluation. *Conflict-freeness* was experimentally disproven for $\delta_{att} > 0.526$. *Naivety*, *SCC recursiveness*, and *INRA* were never fulfilled in the experimental evaluation for any value of δ_{att} used.

For different values of ϵ, the behavior of *At-Count* did not vary significantly (see Figure 48 in the appendix).

Whereas the results for *At-Count* seemed promising, the potentially optimal threshold $\delta_{att} = 0.229$ was unstable when re-tested against edge cases. Thus, the *counting* semantics was declared potentially unsuitable for creating *At-*τ.

At-nsa For *At-nsa*, the maximum number of potentially fulfilled principles in the evaluation was reached for $\delta_{att} \geq 0.62$ for different values of ϵ (see Figure 45 in the appendix). However, those potentially fulfilled principles only included *CF-reinstatement*, *I-maximality*, *directionality*, *modularization*, and *reduct admissibility* (see Figure 16). *Conflict-freeness* or *admissibility* were experimentally disproven for any values of δ_{att} used.

Thus, *nsa* was considered to be potentially unsuitable for creating *At-*τ.

At-hCat For *At-hCat* with $\epsilon = 0.0001$, the maximum number of potentially fulfilled principles was reached for $\delta_{att} \leq 0.09$ in the evaluation (see Figure 17).

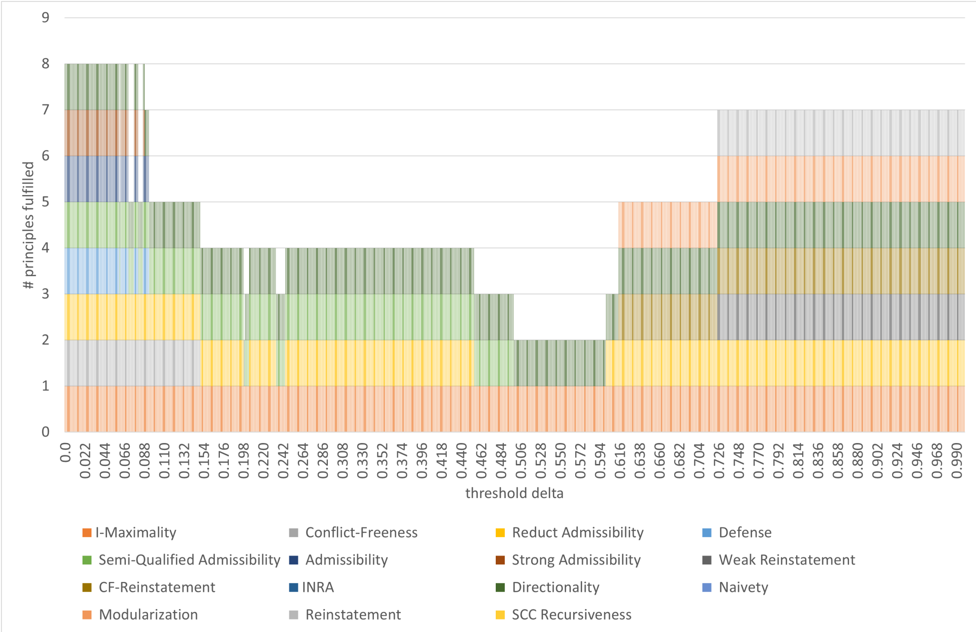

Figure 17: Principles fulfilled for *At-hCat* with $\epsilon = 0.0001$. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

Conflict-freeness could be experimentally disproven for $\delta_{att} > 0.151$, *admissibility* was not fulfilled for $\delta_{att} > 0.061$

For different values of ϵ, the behavior of *At-hCat* did not vary significantly (see Figure 42 in the appendix).

However, a potentially optimal threshold of $\delta_{att} = 0.09$ for *At-hCat* with $\epsilon = 0.0001$ proved to be unstable: For edge-case argumentation frameworks, *admissibility* could not be guaranteed.

4.2.2 Implementing *Re*-τ and *Ar*-τ^{ad}

Whereas the algorithm for implementing *Ar*-τ and *At*-τ required determining δ_{arg} resp. δ_{att} for τ, no absolute thresholds are needed for *Re*-τ and *Ar*-τ^{ad}.

Given an $AF = \langle A, attacks \rangle$, both *Re*-$\tau$ as well as *Ar*-τ^{ad} provide only one extension $E \subseteq A$ per argumentation framework. If the acceptance condition is not met for any argument, then $E = \emptyset$.

As with *Ar*-τ and *At*-τ, an experimental evaluation has been performed to determine the gradual semantics τ potentially suitable for creating *Re*-τ as well as *Ar*-τ^{ad}. We declared a semantics τ as potentially suitable if *admissibility* could not be disproven in the experimental evaluation.

Implementing *Re-*τ Given an argumentation framework $AF = \langle A, attacks \rangle$, the acceptability of an argument $a \in A$ for *Re-*τ is determined by its relative strength: The strength $Deg^{\tau}_{AF}(a)$ has to be greater than the strength of any of its attackers $b \in Att(a)$, s.t. $Deg^{\tau}_{AF}(a) > Deg^{\tau}_{AF}(b)$. How the *Re-*$\tau$-extension is determined, is described in Algorithm 3.

In the experimental evaluation, the following results for *Re-*τ were computed (see Table 4):

Re-nsa*, *Re-hCat*, and *Re-Count For *Re-nsa*, *Re-hCat* and *Re-Count* with $\epsilon = 0.0001$, only *I-maximality* and *directionality* were potentially fulfilled, all other principles were experimentally disproven.

Re-M&T For *Re-M&T*, only *I-maximality*, *directionality* and *CF-reinstatement* were potentially fulfilled, all other principles were experimentally disproven.

Re-Mbs*, *Re-Embs*, and *Re-ITS/Tbs For *Re-Mbs*, *Re-Embs*, *Re-ITS*, and *Re-Tbs* with $\epsilon = 0.0001$ no principle except for *naivety* could be experimentally disproven.

As *admissibility* and *conflict-freeness* was experimentally disproven for *Re-nsa*, *Re-hCat*, *Re-M&T*, and *Re-Count*, only *Mbs*, *Embs*, *ITS*, and *Tbs* were found to be potentially suitable for creating *Re-*τ.

Table 4: Principle-based experimental evaluation of *Re-*τ

Postulates	Ranking-Based Extension Semantics							
	Re-hCat	Re-nsa	Re-Mbs	Re-Embs	Re-Tbs	Re-ITS	Re-Count	Re-M&T
Admissibility	×	×	✓	✓	✓	✓	×	×
Strong Admissibility	×	×	✓	✓	✓	✓	×	×
Semi-Qual. Adm.	×	×	✓	✓	✓	✓	✓	×
Reduct Admissibility	×	×	✓	✓	✓	✓	×	×
Conflict-Freeness	×	×	✓	✓	✓	✓	×	×
Defense	×	×	✓	✓	✓	✓	×	×
Modularization	×	×	✓	✓	✓	✓	×	×
Naivety	×	×	×	×	×	×	×	×
I-Maximality	✓	✓	✓	✓	✓	✓	✓	✓
INRA	×	×	✓	✓	✓	✓	×	×
Reinstatement	×	×	✓	✓	✓	✓	×	×
Weak Reinstatement	×	×	✓	✓	✓	✓	×	×
CF-Reinstatement	×	×	✓	✓	✓	✓	×	✓
Directionality	✓	✓	✓	✓	✓	✓	✓	✓
SCC-Recursiveness	×	×	✓	✓	✓	✓	×	×

Implementing *Ar-*$\tau^{\mathbf{ad}}$ Regarding the *Ar-*τ^{ad} semantics, a variable threshold δ^{AF}_{ad} is determined for every AF individually. Given an argumentation framework $AF = \langle A, attacks \rangle$, an argument $a \in A$ is part of an extension $E \in$*Ar-*$\tau^{ad}(AF)$ with $E \subseteq A$ iff $Deg^{\tau}_{AF}(a) > \delta^{AF}_{ad}$. How the *Ar-*$\tau^{ad}$-extension is determined, is described in Algorithm 4.

Algorithm 3 Determining the *Re*-τ-extension

Input: a directed graph $AF = \langle A, attacks \rangle$,
a gradual semantics τ
Output: $E \subseteq A$ with $E \in$ *Re*-τ(AF)
$ranking \leftarrow$ empty map
for each a in A **do**
  $key \leftarrow a$
  $value \leftarrow Deg^{\tau}_{AF}(a)$
  add $(key, value)$ to $ranking$
end for each
$E \leftarrow$ empty extension
for each a in A **do**
  $attackers \leftarrow Att(a)$
  $argEntry \leftarrow ranking.get(a)$
  $inExt \leftarrow true$
  for each att in $attackers$ **do**
    $attEntry \leftarrow ranking.get(att)$
    if $(attEntry.value \geq argEntry.value)$ **then**
      $inExt \leftarrow false$
    end if
  end for each
  if $inExt$ **then**
    add $entry.key$ to E
  end if
end for each
return E

Algorithm 4 Determining the *Ar*-τ^{ad}-extension

Input: a directed graph $AF = \langle A, attacks \rangle$,
a gradual semantics τ
Output: $E \subseteq A$ with $E \in$ *Ar*-τ $^{ad}(AF)$
$list \leftarrow \{\}$
for each a in A **do**
  compute $Deg^{\tau}_{AF}(a)$ and add to $list$
end for each
$dist \leftarrow$ distinct values from $list$, in descending order
$E \leftarrow$ empty extension
for each d in $dist$ **do**
  $candidates \leftarrow$ all arguments $b \in A$ where $Deg^{\tau}_{AF}(b) = d$
  add all $candidates$ to E
  if E is not admissible **then**
    remove all $candidates$ from E
   return E
  end if
end for each
  return E

As *admissibility* is guaranteed with *Ar-*τ^{ad}, the experimental results looked promising for all gradual semantics τ used (see Table 5).

The *Ar-*τ^{ad} enforces *admissibility* at the cost of removing arguments disrupting it. Thus, the average percentage of accepted arguments for an AF has also been considered to assess the semantics' usefulness in finding a non-empty extension, i.e., a valid point of view in an AF. The results seemed equally promising.

For the argumentation frameworks used in the experimental evaluation, all extension semantics *Ar-*τ^{ad} based on *Mbs*, *Embs*, *ITS*, or *Tbs* returned an extension with, on average, 24.65% of the arguments. The extension semantics *Ar-M&T*^{ad} provided extensions with, on average, 30.2% of the arguments.

The extension semantics *Ar-hCat*^{ad} provided extensions with, on average, 31.2% of the overall arguments. The semantics *Ar-nsa*^{ad} accepted 28.7% and the semantics *Ar-Count*^{ad} accepted, on average, 31.1% of the arguments.

Table 5: Principle-based experimental evaluation of *Ar-*τ^{ad}

	Ranking-Based Extension Semantics							
Postulates	Ar-hCat^{ad}	Ar-nsa^{ad}	Ar-Mbs^{ad}	Ad-Embs^{ad}	Ad-Tbs^{ad}	Ad-ITS^{ad}	Ad-Count^{ad}	Ad-M&T^{ad}
Admissibility	✓	✓	✓	✓	✓	✓	✓	✓
Strong Admissibility	×	×	✓	✓	✓	✓	×	×
Semi-Qual. Adm.	✓	✓	✓	✓	✓	✓	✓	✓
Reduct Admissibility	✓	✓	✓	✓	✓	✓	✓	✓
Conflict-Freeness	✓	✓	✓	✓	✓	✓	✓	✓
Defense	✓	✓	✓	✓	✓	✓	✓	✓
Modularization	×	×	✓	✓	✓	✓	×	✓
Naivety	×	×	×	×	×	×	×	×
I-Maximality	✓	✓	✓	✓	✓	✓	✓	✓
INRA	×	×	✓	✓	✓	✓	×	×
Reinstatement	×	×	✓	✓	✓	✓	×	✓
Weak Reinstatement	×	×	✓	✓	✓	✓	×	✓
CF-Reinstatement	×	×	✓	✓	✓	✓	×	✓
Directionality	✓	×	✓	✓	✓	✓	✓	✓
SCC-Recursiveness	×	×	✓	✓	✓	✓	×	×

5 Principle-Based Evaluation of the Newly Created Semantics

After discussing the implementation details in Chapter 4, this chapter will analyze the most promising newly created extension semantics Ar-τ, At-τ, and Re-τ. We will formally prove that the newly created semantics fulfill the principles declared to be *potentially fulfilled* in the experimental evaluation.

We will analyze why only certain gradual semantics τ, namely *Mbs*, *Embs*, *M&T*, *ITS*, and *Tbs* were found to be potentially suitable for creating σ_{ext_grad}. We will formally prove that for $\tau \in \{$ *hCat*, *Count*, *nsa*$\}$, *admissibility* cannot be guaranteed for Ar-τ, At-τ, and Re-τ. We will also show that no valid stable threshold δ_{arg} or δ_{att} can be found.

Last but not least, we will discuss how the gradual semantics' properties influence the principles fulfilled by the newly created extension semantics Ar-τ, At-τ, and Re-τ. We will offer suggestions for future research regarding the principle-based evaluation of these new semantics.

5.1 Formal Evaluation

In the experimental evaluation in Chapter 4, the gradual semantics $\tau \in \{$*Mbs*, *Embs*, *ITS*, *Tbs*$\}$ delivered the most promising results for creating Ar-τ, At-τ, and Re-τ (see Table 6). Only *naivety* could be experimentally disproven. For Ar-τ and At-τ, the thresholds δ_{arg} and δ_{att} in Table 7 returned the most potentially fulfilled principles.

Table 6: Principle-Based Evaluation of *Ar*/*At*/*Re*-τ with $\tau \in \{$*Mbs*, *Embs*, *Tbs*, *ITS*$\}$ and the *grounded* semantics (*gr*).

	Semantics			
Postulates	Ar-τ	At-τ	Re-τ	gr
Admissibility	✓	✓	✓	✓
Strong Admissibility	✓	✓	✓	✓
Semi-Qualified Admissibility	✓	✓	✓	✓
Reduct Admissibility	✓	✓	✓	✓
Conflict-Freeness	✓	✓	✓	✓
Defense	✓	✓	✓	✓
Modularization	✓	✓	✓	✓
Naivety	×	×	×	×
I-Maximality	✓	✓	✓	✓
INRA	✓	✓	✓	✓
Reinstatement	✓	✓	✓	✓
Weak Reinstatement	✓	✓	✓	✓
CF-Reinstatement	✓	✓	✓	✓
Directionality	✓	✓	✓	✓
SCC-Recursiveness	✓	✓	✓	✓

However, it is important to note that for all argumentation frameworks used in our experimental evaluations, the extensions returned by *Ar/Re/At*-τ with $\tau \in \{$*Mbs*,

Table 7: Thresholds used for *Ar*/*At*-τ with $\tau \in$ {*Mbs*, *Embs*, *Tbs*, *ITS*, *M&T*}.

	Thresholds	
τ	δ_{arg}	δ_{att}
Mbs	0.6181	0.618
Embs	0.5672	0.567
ITS	0.5	0.49999
Tbs	0.5	0.49999
M&T	0.5	-

Embs, *ITS*, *Tbs*} with the potentially optimal thresholds were always equal to the respective *grounded* extension.

Based on the observations of Amgoud and Beuselinck in [6] and our own findings, we will show that for $\tau \in$ {*Mbs*, *Embs*, *ITS*, *Tbs*}, *Ar*-τ, *At*-τ, and *Re*-τ fulfill the properties in Table 6 when using the thresholds defined in Table 7. We will do this by proving that for $\tau \in$ { *Mbs*, *Embs*, *ITS*, *Tbs*}, for any AF, the extension $E_1 \in$ *Ar/Re/At*-$\tau(AF)$ with the optimal thresholds always contains the same arguments as the *grounded* extension $E_2 \in Gr(AF)$ s.t. $E_1 = E_2$.

Ar/Re/At-ITS* and *Ar/Re/At-Tbs For *ITS* and *Tbs*, Amgoud and Beuselinck have shown in [6] that for any non-weighted argumentation graph $AF = \langle A, attacks \rangle$ with $a \in A$, $Deg^{ITS}_{AF}(a)$ resp. $Deg^{Tbs}_{AF}(a)$ are dependent on the relationship between a and the *grounded* extension $Gr(AF)$.

Based on these observations, we can show that the following theorems are true.

Theorem 4. For $\tau \in$ {*Tbs*, *ITS*}, given any AF with $E_2 \in Gr(AF)$ and $E_1 \in$ *Ar*-$\tau(AF)$, $E_1 = E_2$ iff $\delta_{arg} \geq 0.5$.

Proof. Amgoud and Beuselinck have shown that we can differentiate between three groups for $\sigma \in \{ITS, Tbs\}$, given an $AF = \langle A, attacks \rangle$ with $a \in A$:

1. Iff $a \in Gr(AF)$, then $Deg^{\sigma}_{AF}(a)$ converges towards 1 with $Deg^{\sigma}_{AF}(a) > 0.5$.
2. Iff $Gr(AF)$ attacks a, then $Deg^{\sigma}_{AF}(a)$ converges towards 0 with $Deg^{\sigma}_{AF}(a) < 0.5$.
3. Iff a does neither belong to the first nor the second group, then $Deg^{\sigma}_{AF}(a) = \frac{1}{2}$.

The argument a is in the *grounded extension* $E_2 \in Gr(AF)$, if $Deg^{\sigma}_{AF}(a) > 0.5$. Thus, as we have defined that a is in the *Ar*-τ extension $E_1 \in$ *Ar*-$\tau(AF)$, if $Deg_{AF}(a)^{\tau} > \delta_{arg}$, $E_1 = E_2$ for $\delta_{arg} \geq 0.5$, . □

Theorem 5. For $\tau \in$ {*Tbs*, *ITS*}, given any AF with $E_2 \in Gr(AF)$ and $E_1 \in$ *At*-$\tau(AF)$, $E_1 = E_2$ iff $\delta_{att} < 0.5$.

Proof. As both *Tbs* and *ITS* assign the value of 0.5 only to all arguments $a \in A$ which are neither in the *grounded* extension nor attacked by it, a must have at least one attacker $b \in Att(a)$ which belongs to the third group s.t. $Deg^{\tau}_{AF}(b) = Deg^{\tau}_{AF}(a) = \frac{1}{2}$. That means that all arguments not in the *grounded* extension have attackers with a strength greater or equal to $\frac{1}{2}$, whereas all arguments attacking the *grounded* extension have a value below $\frac{1}{2}$. Thus, we can prove that for $\tau \in \{Tbs, ITS\}$, given any AF with $E_2 \in Gr(AF)$ with $E_1 \in At\text{-}\tau(AF)$, $E_1 = E_2$ iff $\delta_{att} < 0.5$. □

Theorem 6. For $\tau \in \{Tbs, ITS\}$, given any AF with $E_2 \in Gr(AF)$ and $E_1 \in Re\text{-}\tau(AF)$, $E_1 = E_2$.

Proof. As all arguments $a \in A$ *not* in the *grounded* extension have attackers $b \in Att(a)$ with a strength greater or equal to $\frac{1}{2}$, there is at least one attacker of a for which $Deg^{\tau}_{AF}(b) \geq Deg^{\tau}_{AF}(a)$. In contrast, all arguments attacking the *grounded* extension have a value below $\frac{1}{2}$. Thus $Re\text{-}\tau(AF)$ coincides with the *grounded* extension $Gr(AF)$, as only arguments from the first group are accepted.

□

Ar/Re/At-Mbs* and *Ar/Re/At-Embs Whereas the thresholds for *Ar/At-*τ with $\tau \in \{Mbs, Embs\}$ listed in Table 7 may seem arbitrary, Amgoud and Beuselinck [6] as well as others [53] have found these values to be quite significant for *Mbs* resp. *Embs*.

For *Ar-Mbs* resp. *At-Mbs*, the threshold δ_{arg} resp. δ_{att} with the highest number of potentially fulfilled principles is connected to the inverse of the so-called *golden ratio*.

Amgoud and Beuselinck have noticed in [6] that the values assigned by *Mbs* are dependent on the Fibonacci sequence for a length n, i.e. $\{F^n\}_{n\geq 0}$ for which

$$F^0 = 0, F^1 = 1 \text{ and } F^n = F^{n-1} + F^{n-2} \text{ for } n > 1.$$

Philippou has proven in [55] that a sequence

$$S^n = \frac{F^n + 1}{F^n}$$

converges towards the *golden ratio*

$$\phi = \frac{1+\sqrt{5}}{2}$$

for $n \to \infty$. In [6], Amgoud and Beuselinck define a new sequence

$$S^n = \frac{F^n}{F^n + 1}$$

with

$$\{S^n\}_{n\geq 1} = \{1, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13}, \frac{13}{21}, \frac{21}{34}, \frac{34}{55}, \frac{55}{89}, \frac{89}{144}, \frac{144}{233}, \frac{233}{377} ...\}$$

and show that for all argumentation frameworks $AF = \langle A, attacks\rangle$, $Deg_{AF}^{Mbs}(a) \in S$ for every $a \in A$. They identify two sub-sequences of S^n, the decreasing sub-sequence S_1 with numbers at odd positions, and the increasing sub-sequence S_2 with numbers at even positions.

$$S_1 = \langle 1, \frac{2}{3}, \frac{5}{8}, \frac{13}{21}, \frac{45}{55}, \frac{89}{144}\frac{233}{377}...\rangle$$

$$S_2 = \langle \frac{1}{2}, \frac{3}{5}, \frac{8}{13}, \frac{21}{34}, \frac{55}{89}, \frac{144}{233}...\rangle$$

Amgoud and Beuselinck observe that both sub-sequences converge towards the same value

$$\lim_{n\to\infty} S_1^n = \lim_{n\to\infty} S_2^n = \frac{1}{\phi} \approx 0.618033$$

with

$$S_2^n < \frac{1}{\phi} < S_1^n,\ \forall n \geq 1.$$

For *Ar-Embs* resp. *At-Embs*, the threshold δ_{arg} and δ_{att} with the highest number of potentially fulfilled principles can be linked to the *Omega Constant* Ω.

The *Omega Constant* Ω is implicitly defined by the following equations connected to *Euler's number*:

$$\Omega e^{\Omega} = 1\ s.t.$$

$$\Omega \sim 0.5671432904.$$

Amgoud and Beuselinck have noticed in [6] that – given an $AF = \langle A, attacks\rangle$ with $a \in A$ – the values $Deg_{AF}^{Embs}(a) \in U$ assigned by *Embs* can be captured by the following equations:

$$U^1 = 1 \text{ and } U^n = e^{-U^{n-1}} \text{ for } n > 1.$$

They identify two sub-sequences of U^n, the decreasing sub-sequence U_{dec} with numbers n at odd positions, and the increasing sub-sequence U_{inc} with numbers n at even positions.

$$U_{dec} = \langle 1, 0.3678, 0.6922, 0.5004, 0.6062, 0.5453, 0.5796...\rangle$$

$$U_{inc} = \langle 0.3678, 0.5004, 0.5433, ...\rangle$$

Amgoud and Beuselinck observe that both sub-sequences converge towards the same value

$$\lim_{n\to\infty} U_{dec} = \lim_{n\to\infty} U_{inc} = \Omega$$

with

$$U_{inc} < \Omega < U_{dec},\ \forall n \geq 1.$$

We will now prove the following theorems.

Theorem 7. Given any AF with $E_2 \in Gr(AF)$ and $E_1 \in Ar\text{-}Mbs(AF)$, $E_1 = E_2$ iff $\delta_{arg} \geq \frac{1}{\phi}$.

Theorem 8. Given any AF with $E_2 \in Gr(AF)$ and $E_1 \in$ *Ar-Embs*(AF), $E_1 = E_2$ iff $\delta_{arg} \geq \Omega$.

Proof. Based on their observations for *Mbs* and *Embs*, Amgoud and Beuselinck have differentiated between three groups of arguments for both semantics. Given an $AF = \langle A, attacks \rangle$ with $a, b \in A$, they show that for $Deg_{AF}^{Mbs}(a) \in S$ and $Deg_{AF}^{Embs}(a) \in U$, the following distinctions can be made:

1. Iff a is defended directly or indirectly by unattacked arguments $b \in Att(a)$, then $Deg_{AF}^{Mbs}(a) \in S_1$ and $Deg_{AF}^{Embs}(a) \in U_{dec}$.
2. Iff a is attacked by an argument b from the first group with $Deg_{AF}^{Mbs}(b) \in S_1$ resp. $Deg_{AF}^{Embs}(b) \in U_{dec}$, then $Deg_{AF}^{Mbs}(a) \in S_2$ resp. $Deg_{AF}^{Embs}(a) \in U_{inc}$.
3. Iff a is neither in the first nor the second group, then $Deg_{AF}^{Mbs}(a) = \frac{1}{\phi}$ and $Deg_{AF}^{Embs}(a) = \Omega$.

The *grounded* extension consists of all unattacked arguments and all arguments that are defended directly or indirectly by unattacked arguments [25]. That means that, given an $AF = \langle A, attacks \rangle$, for any argument $a \in Gr(AF)$, $Deg_{AF}^{Mbs}(a) > \frac{1}{\phi}$ and $Deg_{AF}^{Embs}(a) > \Omega$. Thus, for *Ar-Mbs*, given any AF with $E_2 \in Gr(AF)$ and $E_1 \in$ *Ar-Mbs*(AF), $E_1 = E_2$ iff $\delta_{arg} \geq \frac{1}{\phi}$. For *Ar-Embs*, given any AF with $E_2 \in Gr(AF)$ and $E_1 \in$ *Ar-Embs*(AF), $E_1 = E_2$ iff $\delta_{arg} \geq \Omega$. □

We will also prove the following theorems.

Theorem 9. Given any AF with $E_2 \in Gr(AF)$ and $E_1 \in$ *At-Mbs*(AF), $E_1 = E_2$ iff $\delta_{att} < \frac{1}{\phi}$.

Theorem 10. Given any AF with $E_2 \in Gr(AF)$ and $E_1 \in$ *At-Embs*(AF), $E_1 = E_2$ iff $\delta_{att} < \Omega$.

Proof. All arguments not in the first group have attackers with a strength greater or equal to $\frac{1}{\phi}$ for *Mbs* resp. Ω for *Embs*. Thus, we can prove that for *At-Mbs*, given any AF with $E_2 \in Gr(AF)$ and $E_1 \in$ *At-Mbs*, $E_1 = E_2$ iff $\delta_{att} < \frac{1}{\phi}$. We can also prove that for *At-Embs*, given any AF with $E_2 \in Gr(AF)$ and $E_1 \in$ *At-Embs*, $E_1 = E_2$ iff $\delta_{att} < \Omega$.

□

We can also show that the following theorem is true.

Theorem 11. For $\tau \in \{$*Mbs*, *Embs*$\}$, given any AF with $E_2 \in Gr(AF)$ and $E_1 \in$ *Re-*$\tau(AF)$, $E_1 = E_2$.

Proof. As all arguments $a \in A$ not in the first group have attackers $b \in Att(a)$ with a strength greater or equal to $\frac{1}{\phi}$, there is at least one attacker of a for which $Deg_{AF}^{Mbs}(b) \geq Deg_{AF}^{Mbs}(a)$. The same is true for *Embs* with Ω instead of $\frac{1}{\phi}$. Thus *Re-*$\tau(AF)$ coincides with the *grounded* extension $Gr(AF)$, as only arguments from the first group are accepted. □

On the equivalence with the grounded extension We have shown that for all new extension semantics *Ar-/Re-/At-*τ with $\tau \in \{$*Mbs*, *Embs*, *ITS*, *Tbs*$\}$, the unique extension for any AF contains the same arguments as the *grounded extension*. Naturally, those newly created semantics fulfill the same properties as the *grounded* semantics. However, compared to the *grounded* semantics, they provide the additional benefit of ranking the arguments in the AF.

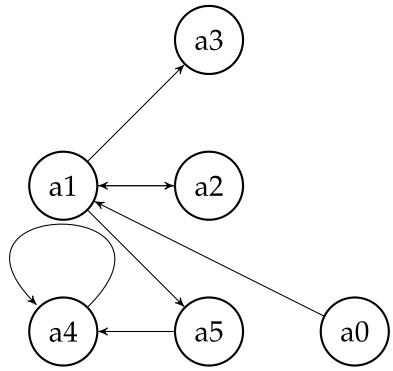

Figure 18: The argumentation framework $AF18$

Table 8: Argument values for $AF18$ (rounded to 7 decimal places) for *Mbs*, *Embs*, *Tbs*, and *ITS* semantics with $\epsilon = 0.0001$

semantics	$a1$	$a2$	$a3$	$a4$	$a5$	$a0$	
Mbs	0.5	0.6666667	0.6666667	0.6	0.6666667	1	
Embs	0.3678794	0.6922006	0.6922006	0.5004735	0.6922006	1	
ITS	0	0.9999951	0.9999951	0.0000923	0.9999951	1	
Tbs	0	0.9999899	0.9999899	0.0001006	0.9999899	1	

Example 20. Given the argumentation framework $AF18$ in Figure 18 and the values in Table 8, we can see that all new extension semantics *Ar/Re/At-*τ with $\tau \in \{$ *Mbs*, *Embs*, *ITS*, *Tbs*$\}$ return the grounded extension $\{a2, a3, a5, a0\}$, given the thresholds defined in Table 7. However, they also all rank the arguments in $AF18$ s.t. $a0 \succ a5 \simeq a3 \simeq a2 \succ a4 \succ a1$ for $\succeq^{\tau}_{AF}$.

As Amgoud and Beuselinck have noted in [6], both *Mbs* and *EMbs* have the additional benefit of providing a more nuanced evaluation of arguments with a broader spectrum of values, compared to *ITS* or *Tbs*.

Ar-M&T For $\tau \in \{$*Ar-M&T*$\}$, only *Ar-*τ returned satisfying results in the experimental evaluation. We will prove the following theorem.

Theorem 12. For $\tau \in \{$*M&T*$\}$, given any AF with an extension $E \in$ *Ar-*$\tau(AF)$, E is admissible for $\delta_{arg} = 0.5$.

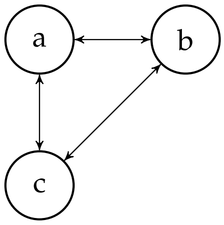

Figure 19: The argumentation framework $AF19$

Proof. Matt and Toni state that for an $AF = \langle A, attacks \rangle$ an argument $a \in A$ is *admissible*, i.e. part of an admissible extension, if $Deg_{AF}^{M\&T}(a) \geq 0.5$ [52]. However, for $\delta_{arg} < 0.5$, $E \in$ *Ar-M&T(AF)* with $a \in E$ might not be admissible, as E might not be conflict-free.

Example 21. Given the $AF19$ in Figure 19, the arguments have the following values for *Ar-M&T*:

- $Deg_{AF}^{M\&T}(a) = Deg_{AF}^{M\&T}(b) = Deg_{AF}^{M\&T}(c) = 0.5.$

As the set $\{a, b, c\}$ is not conflict-free, a threshold $\delta_{arg} < 0.5$ is no optimal threshold for *Ar-M&T*.

However, with $\delta_{arg} \geq 0.5$ *admissibility* is guaranteed. For *M&T*, the set P with $P \subseteq A, a \in P$ denotes a strategy available to the proponent with regard to an argument a. The set O with $O \subseteq A$ denotes a strategy available to the opponent. The degree of acceptability ϕ of P with respect to O is defined by considering the set of attacking arguments s.t.

$$\phi(P, O) = \frac{1}{2}(1 + f(|O_{AF}^{\leftarrow P}|) - f(|P_{AF}^{\leftarrow O}|).$$

Matt has argued in [51] that a value of $\phi(P, O) = 0.5$ means that the reward of the proponent strategy is equal to the reward of an opponent strategy, s.t.

$$f(|O_{AF}^{\leftarrow P}|) = f(|P_{AF}^{\leftarrow O}|).$$

That means if $Deg_{AF}^{M\&T}(a) = 0.5$ there is at least another non-empty admissible extension $E \setminus \{a\}$ attacking a. Thus, to select arguments that are only in one admissible set, δ_{arg} has to be greater or equal to 0.5.

For *Ar-M&T*, the threshold $\delta_{arg} = 0.5$ is stable. Matt and Toni have shown in [52] that for an $AF = \langle A, attacks \rangle$ with an argument $a \in A$ that has k attacks

$$Deg_{AF}^{M\&T}(a) < 1 - \frac{1}{2}(1 - \frac{1}{k+1}).$$

For $k \to \infty$, $Deg_{AF}^{M\&T}(a)$ converges towards 0.5. Thus, for *Ar-M&T* with $\delta_{arg} = 0.5$, *admissibility* will be guaranteed. □

At/Re-M&T* and *At/Ar/Re-nsa Whereas the gradual semantics $\tau \in \{$*Mbs*, *Embs*, *ITS*, *Tbs*$\}$ were found to be suitable for creating *Ar/At/Re*-τ, *At/Re-M&T* and *Ar/At/Re-nsa* were not. We will now show that the reason for this unsuitability of *nsa* and *M&T* is their treatment of self-attacking arguments. We will prove the following theorems.

Theorem 13. Given any AF with $\tau \in \{$*nsa*, *M&T*$\}$ and $E \in$*At*-τ*(AF)*, *admissibility* cannot be guaranteed for E.

Theorem 14. Given any AF with $\tau \in \{$*nsa*$\}$ and $E \in$*Ar*-τ*(AF)*, *admissibility* cannot be guaranteed for E.

Theorem 15. Given any AF with $\tau \in \{$*nsa*, *M&T*$\}$ and $E \in$*Re*-τ*(AF)*, *admissibility* cannot be guaranteed for E.

Proof. For $\tau \in \{$*nsa*, *M&T*$\}$ – given an $AF = \langle A, attacks \rangle$ with $a \in A$ – any self-attacking argument a has a value of 0. As we have defined $\delta_{att} > 0$, this means that when using *At*-τ, an argument a only attacking itself is always accepted because $Deg^{\tau}_{AF}(a) = 0$ is smaller than any value of δ_{att}. Thus, *conflict-freeness* cannot be guaranteed for *At*-τ.

For *Ar-nsa*, given an $AF = \langle A, attacks \rangle$ with $a, b \in A$ – any argument b only attacked by a self-attacking argument a has a value of 1 and is thus accepted, as $\delta_{arg} < 1$. If b is not defended by another accepted argument, the resulting extension is not *admissible* regardless of the value for δ_{arg}.

For *Re*-τ, – given an $AF = \langle A, attacks \rangle$ with $a, b \in A$ – any argument $b \in A$ only attacked by a self-attacking argument a will be accepted, as $Deg^{\tau}_{AF}(a) < Deg^{\tau}_{AF}(b)$. Thus, *admissibility* cannot be guaranteed for *Re*-τ. We will show this with an example.

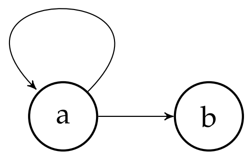

Figure 20: The argumentation framework $AF20$

Example 22. Given the $AF20$ in Figure 20, the arguments have the following values for *M&T* and *nsa*:

- For *M&T*, $Deg^{M\&T}_{AF}(a) = 0$ and $Deg^{M\&T}_{AF}(b) = 0.25$.
- For *nsa*, $Deg^{nsa}_{AF}(a) = 0$ and $Deg^{nsa}_{AF}(b) = 1$.

For *Re-M&T*, *Re-nsa* and *Ar-nsa*, $\{b\}$ is the extension s.t. *admissibility* is not fulfilled. For *At-M&T* and *At-nsa*, $\{a, b\}$ is the extension for any value of $\delta_{att} > 0$ s.t. *conflict-freeness* and *admissibility* are not fulfilled.

□

Ar/At/Re-hCat and Ar/At/Re-nsa The stability of the thresholds with regard to *Ar/At-*τ with $\tau \in$ { *Mbs*, *Embs*, *ITS*, *Tbs*} was implicitly proven by Amgoud and Beuselinck [6]. However, for $\tau \in$ { *Count*, *nsa*, *hCat*} no stable threshold $\delta_{arg} < 1$ resp. $\delta_{att} > 0$ could be found in the experimental evaluation.

We will now prove the following theorems.

Theorem 16. For $\tau \in$ { *nsa*, *hCat*} no stable threshold $\delta_{arg} < 1$ can be found s.t. $E \in$*Ar-*τ*(AF)* is admissible for any AF.

Theorem 17. For $\tau \in$ { *nsa*, *hCat*} no stable threshold $\delta_{att} > 0$ can be found s.t. $E \in$*At-*τ*(AF)* is admissible for any AF.

Theorem 18. For $\tau \in$ { *nsa*, *hCat*}, $E \in$*Re-*τ*(AF)* is not admissible for any AF.

Proof. Given, an $AF = \langle A, attacks \rangle$ with $a, b \in A$ and $(a, b) \in attacks$, for $\tau \in$ { *hCat*,*nsa*} semantics, the equation for computing the strength of all non-self-attacking arguments is

$$Deg^{\tau}_{AF}(a) = \frac{1}{1 + \sum_{b \in Att(a)} Deg^{\tau}_{AF}(b)}.$$

Different from *Mbs*, *Embs*, *ITS*, and *Tbs*, not the strength of the strongest attacker, but the sum of all attacking arguments is considered for *nsa* and *hCat*.

Example 23. Regarding $AF21$ in Figure 21, the argument values given by $\tau \in$ { *nsa*, *hCat*} are:

- $Deg^{\tau}_{AF}(a1) \approx 0.97$,
- $Deg^{\tau}_{AF}(a2) \approx 0.031$, and
- $Deg^{\tau}_{AF}(a3) = ... = Deg^{\tau}_{AF}(a104) \approx 0.618$.

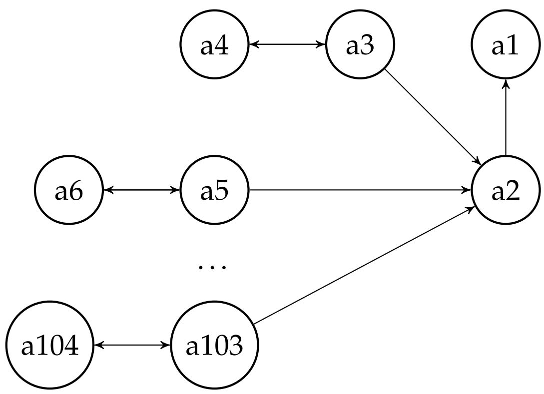

Figure 21: Abstract argumentation framework $AF21$

In this example,

$$Deg^{\tau}_{AF}(a1) = \frac{1}{1 + Deg^{\tau}_{AF}(a2)}$$

s.t. decreasing the strength of $a2$ results in a higher value for $a1$. Adding n more arguments $b \in Att(a2)$ with $Deg^{\tau}_{AF}(b) \approx 0.618$ reduces $a2$. With $n \to \infty$, $Deg^{\tau}_{AF}(a2)$ converges towards 0 and $Deg^{\tau}_{AF}(a1)$ converges towards 1. If $Deg^{\tau}_{AF}(a2) < \delta_{arg} < 0.618$, the resulting extension would consist of $\{a4, a3..., a103, a1\}$ and would not be admissible. If $\delta_{arg} > 0.618$, the resulting extension would consist of $\{a1\}$ and would not be admissible. As only an empty extension would be admissible for *Ar-*τ, $\delta_{arg} > Deg^{\tau}_{AF}(a1)$. As $Deg^{\tau}_{AF}(a1)$ converges towards 1 for $n \to \infty$, a stable threshold $\delta_{arg} < 1$ is not possible for *Ar-hCat* resp. *Ar-nsa*.

When using *At-hCat* resp. *At-nsa*, a stable threshold $\delta_{att} > 0$ is also not possible, as $Deg^{\tau}_{AF}(a2)$ converges towards 0 for $n \to \infty$ and the threshold has to be adjusted s.t. $a1$ is not in the *At-hCat* resp. *At-nsa* extension.

Re-hCat resp. *Re-nsa* also does not produce an admissible extension, as only $a1$ would be accepted, because $Deg^{\tau}_{AF}(a2) < Deg^{\tau}_{AF}(a1)$.

□

Ar/At/Re-Count For *Count*, the overall numbers of defenders and attackers are considered. Given an $AF = \langle A, attacks\rangle$, an argument $a \in A$ is more acceptable if the number of defenders is higher and the number of attackers is lower. As the results of our experimental evaluation have shown, any threshold for *Ar/At-Count* is also unstable.

We will now prove the following theorems.

Theorem 19. Given an AF, for $\tau \in \{$*Count*$\}$ no stable threshold $\delta_{arg} < 1$ can be found s.t. for $E \in$*Ar-*τ*(AF)*, *admissibility* can be guaranteed.

Theorem 20. Given an AF, for $\tau \in \{$*Count*$\}$ no stable threshold $\delta_{att} > 0$ can be found s.t. for $E \in$*At-*τ*(AF)*, *admissibility* can be guaranteed.

Theorem 21. Given an AF, for $\tau \in \{$*Count*$\}$, for $E \in$*Re-*τ*(AF)*, *admissibility* cannot be guaranteed.

Proof. Pu et al. [58] have shown that – given an $AF = \langle A, attacks\rangle$ with $i, j \in A$ – iff $Att(j) \subset Att(i)$, $Deg^{Count}_{AF}(i) < Deg^{Count}_{AF}(j)$.

Example 24. Regarding $AF21 = \langle A, attacks\rangle$ in Figure 21, the argument values given by the *Count* semantics are:

- $Deg^{Count}_{AF}(a1) \approx 0.998$,
- $Deg^{Count}_{AF}(a2) \approx 0.116$, and
- $Deg^{Count}_{AF}(a3) = ...Deg^{Count}_{AF}(a104) \approx 0.983$.

In our example, adding n new argument pairs $x, y \in A$ with $(x, y), (y, x), (y, a2) \in attacks$ will decrease the strength of $a2$. As the number of attackers of $a2$ increases, the strength of $Deg_{AF}^{Count}(a1)$ decreases. With $n \to \infty$, $Deg_{AF}^{Count}(a2)$ also converges towards 0 and $Deg_{AF}^{Count}(a1)$ converges towards 1.

Similar to *hCat* and *nsa*, this makes it impossible to define a stable threshold δ_{arg} resp. δ_{att} for *Ar-Count* resp. *At-Count*. *Re-Count* also does not produce an admissible extension, as only $a1$ would be accepted, because $Deg_{AF}^{Count}(a2) < Deg_{AF}^{Count}(a1)$.

□

5.2 Discussion

As the experimental evaluation in Chapter 4 has shown, not all gradual semantics τ provided satisfying results for the new extension-based semantics *Ar-*τ, *At-*τ, and *Re-*τ.

Table 9: Fulfillment of postulates for different semantics.

	Postulates															
Sem.	SC	CT	SCT	QP	DP	+AB	↑DB	↑AB	AvsFD	CN	VP	DDP	+DB	CP	⨁DB	RN
hCat	×	✓	✓	×	✓	✓	✓	✓	×	✓	✓	×	×	×	×	✓
Mbs	×	✓	×	✓	×	×	×	×	✓	×	✓	×	×	×	×	✓
Embs	×	✓	×	✓	×	×	×	×	✓	×	✓	×	×	×	×	✓
Tbs	×	✓	×	✓	×	×	×	×	✓	×	×	×	×	×	×	✓
ITS	×	✓	×	✓	×	×	×	×	✓	×	×	×	×	×	×	✓
Count	×	✓	✓	×	✓	✓	✓	✓	×	✓	✓	×	×	×	×	✓
M&T	✓	×	×	×	×	✓	×	×	✓	×	✓	×	×	×	×	×
nsa	✓	×	×	×	×	×	×	×	×	✓	×	×	×	×	×	×
Grd.	×	✓	×	✓	×	×	×	×	✓	×	×	×	×	×	×	?

This chapter contains an analysis of how the properties fulfilled by τ influence its suitability for creating *Ar-*τ, *At-*τ, and *Re-*τ. The focus will be on *admissibility*. For comparison, a principle-based evaluation of the *grounded semantics* as a ranking-based semantics [21] – using a degenerate ranking of either *accepted* or *rejected* – is considered (see Table 9).

As only a limited number of gradual semantics were studied, the analysis will concentrate on the principles fulfilled by either one of those semantics. As *DDP* as well as *+DB* and ⨁ *DB* are not satisfied by any of the gradual semantics used in this thesis, we have neglected these properties for our analysis.

Terminology Given an $AF = \langle A, attacks \rangle$ with $a, b \in A$, a semantic property $prop_{sem1}$ is called *compatible* with another property $prop_{sem2}$ iff, when $prop_{sem1}$ states that $a \succ_{AF} b$, then $prop_{sem2}$ does not provide a ranking for which $b \succ_{AF} a$ [34].

Incompatible principles Whereas other gradual semantics deliver very promising results, for $\tau \in \{$ *M&T*, *hCat*, *nsa*, *Count*$\}$, *admissibility* could not be guaranteed for *At*-τ, and *Re*-τ. For $\tau \in$ { *hCat*, *nsa*, *Count*}, no stable thresholds δ_{arg} and δ_{att} could be found for *Ar*-τ and *At*-τ.

The following properties can be shown to be responsible for an unsuitability of τ for *Ar*-τ, *At*-τ or *Re*-τ.

SC and *At*/*Re*-τ With classical extension-based semantics, self-attacking arguments are always rejected [19]. However, self-attacking arguments are not necessarily ranked lower than other rejected ones for classical semantics, so *SC* is not fulfilled.

When creating *Ar*-τ, gradual semantics such as *M&T* fulfilling *SC* can be used with satisfactory results. If gradual semantics τ fulfilling *SC* are used for the creation of *At*-τ or *Re*-τ, however, *admissibility* is not guaranteed.

Example 25. For semantics τ fulfilling *SC*, self-attacking arguments are ranked lower than all other arguments s.t. for any $AF = \langle A, attacks\rangle$ with $a, b \in A$, if $(a, a) \notin attacks, (b, b) \in attacks$, then $a \succ^{\tau}_{AF} b$ [2] (see $AF20$ in Figure 20).

However, that means for *Re*-τ that any non-self-attacking argument a that is only attacked by a self-attacking argument b would be accepted, as $Deg^{\tau}_{AF}(b) < Deg^{\tau}_{AF}(a)$. For *At*-$\tau$, given that $Deg^{\tau}_{AF}(b) < \delta_{att}$, a would be accepted as well. Thus, *admissibility* could not be guaranteed.

SCT and *Ar*/*At*/*Re*-τ For $\tau \in$ { *Count*, *nsa*, *hCat*} no stable threshold $\delta_{arg} < 1$ resp. $\delta_{att} > 0$ could be found in the experimental evaluation. We will now show that any gradual semantics fulfilling *SCT* is not suitable as a basis for *Ar*-τ or *At*-τ, as a stable threshold might not be guaranteed.

Given an $AF = \langle A, attacks\rangle$ with $a, b \in A$, if a semantics τ fulfills *SCT*, then b is ranked higher than a if the group of attackers of a is larger or has arguments more acceptable than b. With regard to the semantics τ, we have defined that given an $AF = \langle A, attacks\rangle$ and $a \in A$, $Deg^{\tau}_{AF}(a) \in [\beta, 1]$.

Example 26. For an $AF = \langle A, attacks\rangle$ with $a1, a2 \in A$ and $(a2, a1) \in attacks$ – like the $AF21$ in Figure $AF21$ – for any τ fulfilling *SCT* the addition of n arguments $j_n \in A$ with $(j_n, a2) \in attacks$ will result in a decrease of $Deg^{\tau}_{AF}(a2)$, as the strength of $Att(a2)$ increases. Simultaneously, this will lead to an increase in $Deg^{\tau}_{AF}(a1)$, as the strength of $Att(a2)$ decreases.

However – if all j_n are rejected with regard to the threshold condition for *At*-τ resp. *Ar*-τ – an extension for *At*-τ or *Ar*-τ would only consists of $\{a1\}$, if $\delta_{arg} < Deg^{\tau}_{AF}(a1)$ resp. $\delta_{att} > Deg^{\tau}_{AF}(a2)$. As $a1$ is not defended by the extension, admissibility would not be guaranteed. Thus, if $Deg^{\tau}_{AF}(a1)$ converges towards the maximum strength value 1 and $Deg^{\tau}_{AF}(a2)$ converges towards the minimum strength value β for τ, for $n \to \infty$, no stable $\delta_{arg} < 1$ resp. $\delta_{att} > \beta$ can be found.

Re-τ definitely does not satisfy *admissibility* for any gradual semantics τ satisfying *SCT*, as $Deg^{\tau}_{AF}(a1) > Deg^{\tau}_{AF}(a2)$ for $n \to \infty$. The resulting extension would not be admissible as only $a1$ would be accepted for *Re-*τ.

Blümel and Thimm [20] also find *SCT* incompatible with classical admissibility semantics.

CN, RN and *Ar*/*At*/*Re*-τ For gradual semantics τ that fulfill *Counting (CN)* and *Reinforcement (RN)*, a stable threshold δ_{arg} resp. δ_{att} might also not be found for *Ar-*τ resp. *At-*τ. It is important to show that *CN* and *RN* might have that effect, as *nsa* is a semantics that does not satisfy *SCT* and nevertheless has no stable threshold.

When *CN* is fulfilled by τ, each non-zero attacker decreases the strength of its target (see Chapter 2.2 for the formal definition). When *RN* is fulfilled by τ, then increasing the strength of an attacker should lead to a decrease in strength for the attacked argument (see Chapter 2.2 for the formal definition).

We will show this potential instability of thresholds for any semantics τ that fulfill *CN* and *RN*.

Example 27. For any semantics τ satisfying *CN*, given an $AF = \langle A, attacks \rangle$ with $a1, a2 \in A$ and $(a2, a1) \in attacks$ – like the $AF21$ in Figure $AF21$ – the addition of n non-zero arguments $j_n \in A$ with $(j_n, a2) \in attacks$ will result in a decrease of $Deg^{\tau}_{AF}(a2)$, as the number of non-zero attackers of $a2$ increases. If τ also fulfills *RN*, decreasing the strength of $a2$ increases the strength of $a1$.

However – if all j_n are rejected with regard to the threshold condition for *At-*τ resp. *Ar-*τ – an extension for *At-*τ or *Ar-*τ would only consists of $\{a1\}$ and not be admissible, if $\delta_{arg} < Deg^{\tau}_{AF}(a1)$ resp. $\delta_{att} > Deg^{\tau}_{AF}(a2)$. Thus, if $Deg^{\tau}_{AF}(a1)$ converges towards the maximum strength value 1 and $Deg^{\tau}_{AF}(a2)$ converges towards the minimum strength value β for τ, for $n \to \infty$, no stable $\delta_{arg} < \alpha$ resp. $\delta_{att} > \beta$ can be found.

Re-τ definitely does not satisfy *admissibility* for any gradual semantics τ satisfying *CN* and *RN*, as $Deg^{\tau}_{AF}(a1) > Deg^{\tau}_{AF}(a2)$ for $n \to \infty$. The resulting extension would not be admissible as only $a1$ would be accepted for *Re-*τ.

Interestingly, *CN* is also violated by *grounded*, *stable*, *preferred*, and *complete* semantics as already shown by Amgoud et al. [8]. As such, it seems incompatible with the principle of *admissibility* in general.

CP and *Ar*/*At*/*Re*-τ *CP* is not satisfied by any of the gradual semantics used in this thesis. However, others, such as [20], have argued that the property is incompatible with *admissibility* and not fulfilled by any classical extension-based semantics [8].

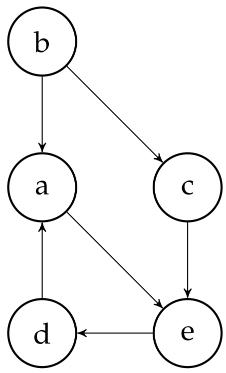

Figure 22: The argumentation framework $AF22$

Example 28. If a *gradual semantics* τ fulfilling *CP* would have been used for $AF22$ in Figure 22, then d would have been ranked higher than e ($d \succ^{\tau}_{AF} e$), as $|Att(e)| > |Att(d)|$. Thus, $\{b, d\}$ would be the *Re-*τ extension for any τ fulfilling *CP*, and thus not be admissible. Depending on the values used for δ_{arg} resp. δ_{att}, $\{b, d\}$ could also be the extension for *Ar-*τ resp.*At-*τ, so *admissibility* would not be fulfilled. In contrast, $e \succ d$ for *grounded*, *preferred* and *complete* semantics.

Compatible principles The properties *CN*, *RN*, *SC*, as well as *SCT* were shown to be responsible for the unsuitability of specific gradual semantics τ for *Ar-*, *At-*, and *Re-*τ. In contrast, gradual semantics τ fulfilling the following principles, might return satisfying results for *Ar-*, *At-*, and *Re-*τ:

AvsFD and *Ar*/*At*/*Re-*τ As others such as [34] have noticed, *hCat* and *Count* as well as *nsa* do not satisfy *AvsFD*. The fulfillment of *AvsFD* by a gradual semantics τ, however, is important for the successful creation of the *Ar-*, *At-*, and *Re-*τ semantics.

If a gradual semantics τ does not satisfy *AvsFD*, arguments defended by unattacked arguments might be ranked higher than those attacked by unattacked arguments. This defies the principle of *defense* resp. *admissibility*.

Example 29. We will show that a semantics not fulfilling *AvsFD* might not guarantee *admissibility* for *Ar-*τ. Given the $AF23$ from Figure 23, the values of a, b for $\tau \in \{$*nsa*, *Count*, *hCat*$\}$ are:

- $Deg^{Count}_{AF}(b) = 0.775$ resp. $Deg^{hCat}_{AF}(b) = Deg^{nsa}_{AF}(b) = 0.5$, and
- $Deg^{Count}_{AF}(a) = 0.303$ resp. $Deg^{hCat}_{AF}(a) = Deg^{nsa}_{AF}(a) = 0.333$.

That means for $\tau \in \{$*nsa*, *Count*, *hCat*$\}$ $b \succ a$.

For any semantics τ not fulfilling *AvsFD*, b is at least as acceptable as a. That means that, depending on the threshold δ_{arg} used, b might be accepted for *Ar-*τ. As b is undefended, *admissibility* cannot be guaranteed.

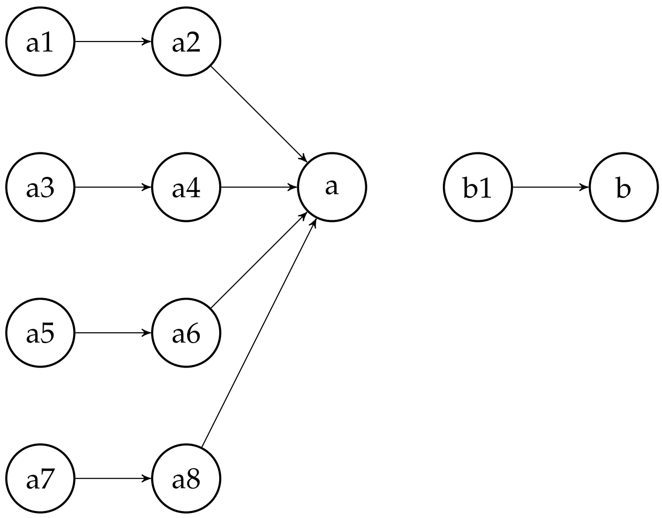

Figure 23: The argumentation framework AF23

In contrast, $a \succ b$ for classical extension-based semantics such as *grounded* semantics, as a is defended and part of the *grounded* extension, whereas b is not.

VP and *Ar*/*At*/*Re*-τ Gradual semantics τ fulfilling *VP* can be used for the creation of *Ar*-, *At*-, or *Re*-τ, as $\tau \in \{$*Mbs*, *Embs*$\}$ show.

Nevertheless, *VP* does not have to be fulfilled for a gradual semantics τ to be suitable for the creation of *Ar*-, *At*-, or *Re*-τ. For instance, it is neither fulfilled by *ITS* nor *Tbs*. Interestingly, neither *grounded*, nor *preferred*, *stable* nor *complete* semantics fulfill *VP*, as defended attacked arguments might have the same status as unattacked arguments [2, 3]. However, classical semantics fulfill a weaker form of *VP*, called *weak void precedence (WVP) [62, 34]*. Given any $AF = \langle A, attacks \rangle$ with $a, b \in A$, *Att*$(a) = \emptyset$ and *Att*$(b) \neq \emptyset$, *WVP* is satisfied iff $a \succeq^{\sigma}_{AF} b$.

Regarding *Ar*-, *At*-, and *Re*-τ, even though fulfilling *VP* is not necessary for τ, semantics not fulfilling *VP* such as *nsa* might also rank arguments which are not part of any admissible extension as high as unattacked arguments s.t. *admissibility* cannot be guaranteed.

Example 30. For *nsa*, arguments attacked only by self-attacking arguments have a value of 1. Given the argumentation framework $AF20$ in Figure 20, $\{b\}$ is the extension for *Ar-nsa* for any value of δ_{arg}, even though it is undefended.

DP and *Ar*/*At*/*Re*-τ *DP* is not fulfilled by any of the gradual semantics τ deemed suitable for the creation of *Ar*/*At*/*Re*-τ.

However, *DP* does not contradict concepts of admissibility-based semantics per se. As Blümel and Thimm [20] have argued, *DP* incorporates ideas of *defense* known from classical admissibility semantics.

Classical semantics, such as *grounded* semantics, do not fulfill *DP* because defended arguments are not necessarily ranked higher than undefended ones. Nevertheless, a weaker form of *DP* is fulfilled: Iff for any $AF = \langle A, attacks \rangle$ with $a, b \in A$, $|Att(a)| = |Att(b)|$, but b is only attacked by non-attacked arguments, then $a \succeq^{\sigma}_{AF} b$ [2].

CT and *Ar*/*At*/*Re*-τ Based on our findings, gradual semantics τ fulfilling *CT* or not fulfilling it can both be used for the creation of *Ar*-τ. For *Mbs*, *Embs*, *Tbs*, *grounded*, and *ITS* semantics, *CT* is fulfilled (see Table 9) whereas for *M&T* semantics it is not.

However, any τ not fulfilling *CT* cannot be used for the creation of *At*- or *Re*-τ semantics, as *admissibility* cannot be guaranteed. Given an $AF = \langle A, attacks \rangle$ with $a, b \in A$, an argument a with a group of attackers at least as large and acceptable as b might be ranked lower than b, if *CT* is not fulfilled.

Example 31. In the argumentation framework AF20 in Figure 20, even though $Att(a) \succeq^{M\&T}_{AF} Att(b)$, $b \succ^{M\&T}_{AF} a$. Thus, the extension $E \in$ *Re*-$M\&T(AF)$ consists of $\{b\}$, even though b is undefended by E. If $\delta_{att} > 0$, b is also accepted for *Ar*-$M\&T(AF)$. Thus, *admissibility* cannot be guaranteed.

QP and *Ar*/*At*/*Re*-τ Based on our findings, gradual semantics τ fulfilling *QP* or not fulfilling it can both be used for the creation of *Ar*-τ. For *Mbs*, *Embs*, *Tbs*, *grounded*, and *ITS* semantics, *QP* is fulfilled (see Table 9) whereas for *M&T* semantics it is not. Generally, gradual semantics τ fulfilling *QP* seem to deliver more promising results for *Ar*-τ regarding the number of principles fulfilled.

However, when it comes to *At*- or *Re*-τ, gradual semantics τ not satisfying *QP* might not be suitable, as *admissibility* might not be guaranteed.

Given an $AF = \langle A, attacks \rangle$ with $a, b \in A$, if a gradual semantics τ does not fulfill *QP*, then $a \succeq_{\tau} b$ is possible, even if a has attackers ranked higher than any attacker of b. Thus, neither $E \in$*At*-$\tau(AF)$ nor $E \in$*Re*-$\tau(AF)$ can be guaranteed to be admissible.

Example 32. Given the argumentation framework AF24 in Figure 24, the values for the *M&T* semantics are

- $Deg^{M\&T}_{AF}(a3) \approx 0.167$, and
- $Deg^{M\&T}_{AF}(a1) = Deg^{M\&T}_{AF}(a2) = Deg^{M\&T}_{AF}(a0) = 0.25$.

Even though $Att(a2) \succ_{M\&T} Att(a2)$, $a2 \simeq_{M\&T} a1$ for *M&T*. Thus, the extension $E \in$ *Re*-$M\&T(AF)$ consists of $\{a1\}$, even though it is undefended by it.

Likewise, the extension $E \in$ *At*-$M\&T(AF)$ might not be admissible depending on the value used for δ_{att}

For semantics fulfilling *QP* such as *Mbs*,

$$a2 \simeq_{Mbs} a1 \simeq_{Mbs} a3 \simeq_{Mbs} a0.$$

Thus, no arguments are accepted for *Re*-$Mbs(AF)$ resp. *At*-$Mbs(AF)$.

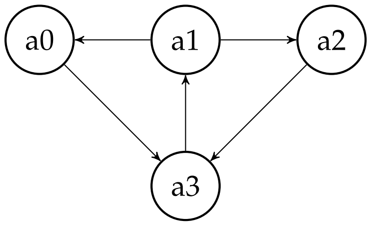

Figure 24: The argumentation framework AF24

↑*DB*/↑*AB* and *Ar*/*At*/*Re*-τ Gradual semantics fulfilling ↑*DB* or ↑*AB* – such as *hCat* and *Count* – do not seem to be suitable for the creation of *Ar*-, *At*-, or *Re*-τ, based on our findings.

However, fulfilling those properties is not incompatible with concepts of Dung-style extension-based semantics. Given an $AF = \langle A, attacks \rangle$, with $a \in A$, *Mbs*, *Embs* as well as *ITS* and *Tbs* fulfill a weaker form of ↑*DB resp.* ↑*AB*, s.t. increasing the length of the defense branch (resp. the attack branch) of a deteriorates or does not affect (resp. improves or does not affect) the ranking of a.

+AB and *Ar*/*At*/*Re*-τ *+AB* is neither fulfilled by *grounded* semantics nor by *Mbs*, *Embs*, *Tbs* or *ITS*. However, even though *M&T* fulfills *+AB*, it can be used for creating new *Ar*-τ.

Delobelle states in [34] that even though admissible semantics like *grounded* semantics do not fulfill *+AB*, the property does not contradict *admissibility* per se. He suggests that a weaker version of *+AB* is fulfilled by *grounded* semantics s.t. given an $AF = \langle A, attacks \rangle$, with $a \in A$, iff σ fulfills $+AB$, then the addition of an attack branch to any argument a deteriorates *or does not affect* the ranking of a.

On the equivalence with *grounded* semantics Our findings that using the gradual semantics $\tau = \{Mbs, Embs, Tbs, ITS\}$ as a basis for *Ar, Re, At*-τ results in an extension with the same arguments as the *grounded* extension, confirms the claim of Amgoud and Beuselinck [6] that those semantics are equivalent to the *grounded* semantics regarding flat graphs.

Looking at the properties fulfilled by those semantics, the fulfillment of the properties QP, $AvsFD$ as well as CT and the non-fulfillment of CN, SC, SCT or CP might be responsible for this equivalence with the *grounded* extension.

5.3 Suggestions for Future Research

While we formally evaluated the new semantics σ_{ext_grad} in this chapter, several research questions still need to be explored and could be investigated in future studies.

Evaluation of principles Not all principles fulfilled by the newly created semantics could be proven in a principle-based evaluation. For *Ar-M&T*, any of the other potentially fulfilled principles besides *admissibility* as observed in Chapter 4.2.1 still have to be proven.

Analysis of semantics Whereas we conducted an extensive experimental evaluation, not all newly created semantics were evaluated. The properties fulfilled for $Ar\text{-}\tau^{ad}$ with $\tau \in \{$*Tbs, ITS, Mbs, Embs, Count, M&T, hCat, nsa*$\}$ were not formally proven. As the experimental results for the $Ar\text{-}\tau^{ad}$ seemed to be promising for *all* gradual semantics τ explored, the properties of the $Ar\text{-}\tau^{ad}$ semantics should be evaluated in future studies.

Ranking- and extension-based principles As mentioned in Chapter 3, the compatibility of ranking-based with extension-based semantics has been discussed in existing research. Blümel and Thimm [20], for instance, introduced σ-*compatibility* to determine the compatibility of a given ranking-based semantics τ with an extension-based semantics σ.

This chapter and most studies have focused on the compatibility between properties of ranking-based semantics and *admissibility*. However, considering our principle-based evaluation of *Ar-*τ, *At-*τ, and *Re-*τ, the compatibility between ranking-based properties and other extension-based principles such as *directionality* or *reinstatement* could be analyzed as well in future studies.

6 Conclusion

In this thesis, we have created new extension semantics σ_{ext_grad} based on different gradual semantics $\tau \in \{$*Tbs*, *ITS*, *Mbs*, *Embs*, *Count*, *M&T*, *hCat*, *nsa*$\}$ – thus bridging the gap between extension- and ranking-based semantics. The new extension semantics Ar-τ, At-τ, Re-τ as well as Ar-τ^{ad} were formally defined and evaluated in an experimental evaluation. As there are currently no detailed studies on the computational complexity of the gradual semantics τ used, the complexity of Ar-τ, At-τ, Re-τ, and Ar-τ^{ad} semantics could not be explored. However, the new semantics were formally analyzed for principles fulfilled.

When using gradual semantics fulfilling *SC* such as *M&T* or *nsa*, *admissibility* could not be guaranteed for *At*-τ. For gradual semantics τ fulfilling *SCT* or *RN* and *CN* such as *hCat*, *nsa*, or *Count*, *admissibility* could not be ensured for *Ar/At/Re*-τ. In contrast, the gradual semantics fulfilling QP, $AvsFD$ and CT and not fulfilling CN, SC, SCT and CP – such as *Tbs*, *ITS*, *Mbs*, and *Embs* – returned satisfying results for Ar-τ, At-τ, and Re-τ. However, the resulting extensions proved equivalent to the *grounded* extension.

Outlook In the future, more extension semantics based on gradual semantics could be created. Using other gradual semantics τ as a basis for σ_{ext_grad} could be explored. Especially, a gradual semantics fulfilling *adm-compatibility* – such as the semantics $\succeq_{ser}$ [20] – sounds promising in this regard.

Whereas this study has focused on creating semantics that fulfill *admissibility*, new extension-based semantics based on gradual semantics satisfying weaker forms of *admissibility* could also be explored. The more nuanced nature of gradual semantics would also allow for a relaxation of the *conflict-freeness* principle by completely disregarding conflicts in an extension under a certain threshold β, as suggested by Dunne et al. [40] with the notion of an *inconsistency budget*.

The procedure of creating new extension semantics based on gradual semantics used in this thesis could be applied to other types of argumentation frameworks as well, such as bipolar [5] or weighted argumentation frameworks [28, 27]. However, the gradual semantics deemed suitable for this endeavor would have to be explored, and the conditions for acceptance would have to be redefined.

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7 Appendix

Principle-Based Evaluation of nsa We will prove by counterexample that the *nsa* semantics does not fulfill +*AB*, *VP*, *DP*, ↑*DB*, *QP*, *AvsFD*, and ↑*AB*.

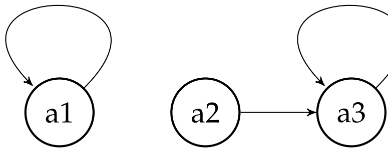

Figure 25: The abstract argumentation framework $AF25$

Theorem 22. The *nsa* semantics does not fulfill +*AB*.

Proof. With $AF25$ in Figure 25, we can show that +*AB* is not fulfilled for *nsa*. For $AF25$, the argument values given by the *nsa semantics* for $\epsilon = 0.0001$ are

- $Deg^{nsa}_{AF}(a1) = Deg^{nsa}_{AF}(a3) = 0$, and
- $Deg^{nsa}_{AF}(a2) = 1$.

This results in the ranking $\succeq^{nsa}_{AF}$: $a2 \succ a3 \simeq a1$.

The principle +*AB* states that $a1$ should be more acceptable than $a3$ because $a3$ has an attack branch while $a0$ has none. However, the *nsa* considers $a1$ and $a3$ as equally acceptable. Thus, *nsa* does not fulfill +*AB*.

□

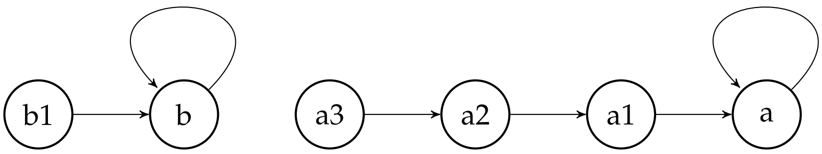

Figure 26: The abstract argumentation framework $AF26$

Theorem 23. The *nsa* semantics does not fulfill ↑*AB*.

Proof. With $AF26$ in Figure 26, we can show that ↑*AB* is not fulfilled for *nsa*. For $AF26$, the argument values given by the *nsa semantics* for $\epsilon = 0.0001$ are

- $Deg^{nsa}_{AF}(a) = Deg^{nsa}_{AF}(b) = 0$,
- $Deg^{nsa}_{AF}(a2) = 0.5$,
- $Deg^{nsa}_{AF}(a1) \approx 0.67$, and
- $Deg^{nsa}_{AF}(a3) = Deg^{nsa}_{AF}(b1) = 1$.

This results in the ranking $\succeq^{nsa}_{AF}$: $a3 \simeq b1 \succ a1 \succ a2 \succ a \simeq b$.

The principle *↑AB* states that b should be more acceptable than a because a has a longer attack branch. However, the semantics considers a and b as equally acceptable. Thus, *nsa* does not fulfill *↑AB*.

□

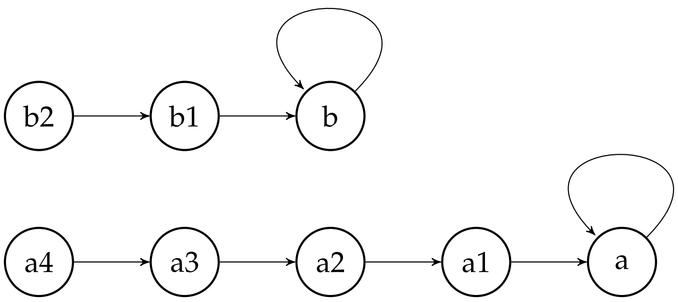

Figure 27: The abstract argumentation framework $AF27$

Theorem 24. The *nsa* semantics does not fulfill *↑DB* or *QP*.

Proof. With $AF27$ in Figure 27, we can show that *↑DB* and *QP* are not fulfilled for *nsa*. For $AF27$, the argument values given by the *nsa semantics* for $\epsilon = 0.0001$ are

- $Deg^{nsa}_{AF}(a) = Deg^{nsa}_{AF}(b) = 0$,
- $Deg^{nsa}_{AF}(a3) = Deg^{nsa}_{AF}(b1) = 0.5$,
- $Deg^{nsa}_{AF}(a1) = 0.6$,
- $Deg^{nsa}_{AF}(a2) \approx 0.67$, and
- $Deg^{nsa}_{AF}(a4) = Deg^{nsa}_{AF}(b2) = 1$.

This results in the ranking $\succeq^{nsa}_{AF}$: $a4 \simeq b2 \succ a2 \succ a1 \succ a3 \simeq b1 \succ a \simeq b$.

The principle *↑DB* states that b should be strictly more acceptable than a because a has a longer defense branch. However, the semantics considers a and b as equally acceptable. Thus, *nsa* does not fulfill *↑DB*.

The principle *QP* states that b should be strictly more acceptable than a because $a1$ is strictly more acceptable than $b1$. However, the semantics considers a and b as equally acceptable. Thus, *nsa* does not fulfill *QP*. □

Theorem 25. The *nsa* semantics does not fulfill *VP*.

Proof.

Example 33. Given the $AF28$ in Figure 28, the arguments have the following values for *nsa*:

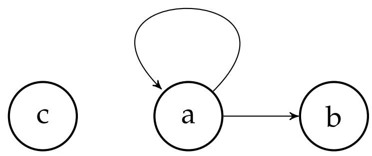

Figure 28: The argumentation framework $AF28$

- $Deg^{nsa}_{AF}(c) = Deg^{nsa}_{AF}(b) = 1$, and
- $Deg^{nsa}_{AF}(a) = 0$.

As the unattacked argument c is ranked as high as the argument b only attacked by the self-attacking argument a, *VP* is not satisfied.

□

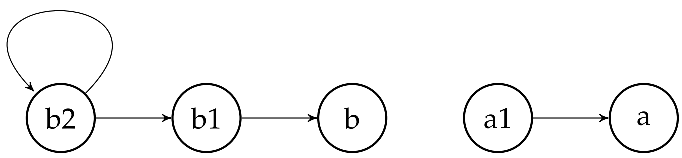

Figure 29: The abstract argumentation framework $AF29$

Theorem 26. The *nsa* semantics does not fulfill *DP*.

Proof. With $AF29$ in Figure 29, we can show that *DP* is not fulfilled for *nsa*. For $AF29$, the argument values given by the *nsa semantics* for $\epsilon = 0.0001$ are

- $Deg^{nsa}_{AF}(a) = Deg^{nsa}_{AF}(b) = 0.5$,
- $Deg^{nsa}_{AF}(b2) = 0$, and
- $Deg^{nsa}_{AF}(a1) = Deg^{nsa}_{AF}(b1) = 1$.

This results in the ranking $\succeq^{nsa}_{AF}$: $a1 \simeq b1 \succ a \simeq b \succ b2$.

The principle *DP* states that b should be strictly more acceptable than a because a is not defended, whereas b is, and they both have the same number of attackers. However, the semantics considers a and b as equally acceptable. Thus, *nsa* does not fulfill *DP*. □

Theorem 27. The *nsa* semantics does not fulfill *AvsFD*.

Proof. With $AF30$ in Figure 30, we can show that *AvsFD* is not fulfilled for *nsa*. For $AF30$, the argument values given by the *nsa semantics* for $\epsilon = 0.0001$ are

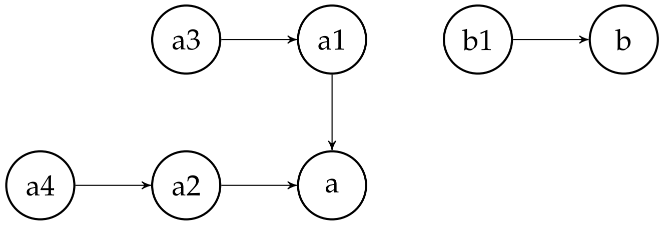

Figure 30: The abstract argumentation framework $AF29$

- $Deg^{nsa}_{AF}(a) = Deg^{nsa}_{AF}(b) = Deg^{nsa}_{AF}(a2) = Deg^{nsa}_{AF}(a1) = 0.5$, and
- $Deg^{nsa}_{AF}(a3) = Deg^{nsa}_{AF}(a4) = Deg^{nsa}_{AF}(b1) = 1$.

This results in the ranking $\succeq^{nsa}_{AF}$: $a1 \simeq b1 \succ a \simeq b \succ b2$.

The principle *AvsFD* states that a should be strictly more acceptable than b because a has only defense branches, whereas b has one direct attacker and no defense branches. However, the semantics considers a and b as equally acceptable. Thus, *nsa* does not fulfill *AvsFD*. □

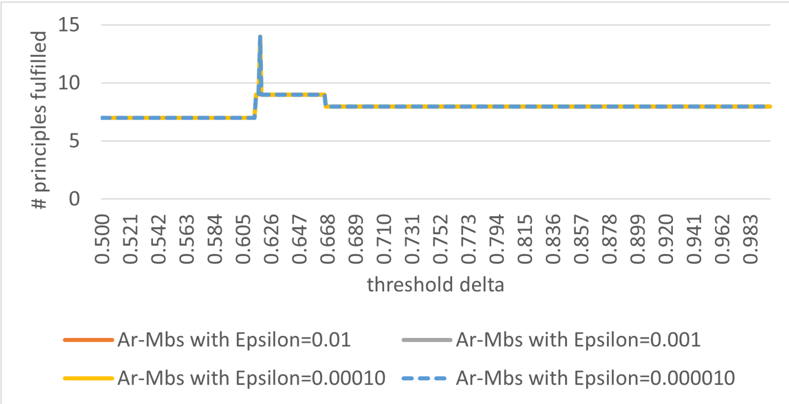

Figure 31: Non-detailed threshold evaluation using *Ar-Mbs*. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

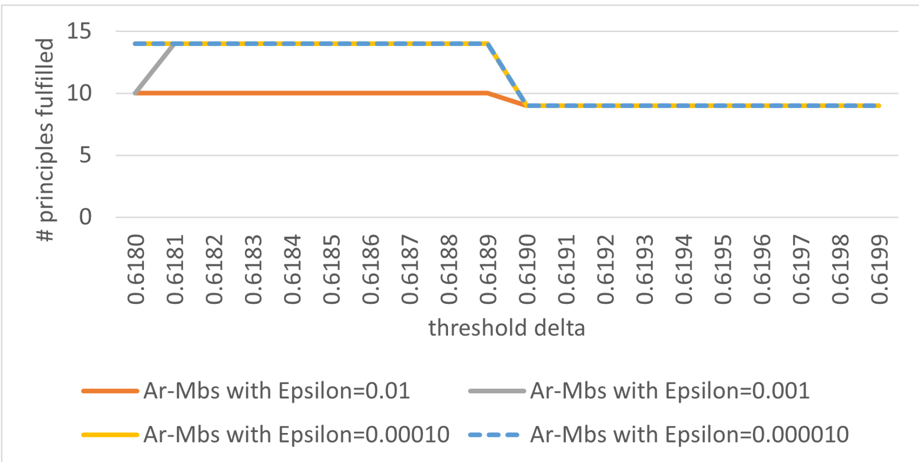

Figure 32: Detailed threshold evaluation using *Ar-Mbs*. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

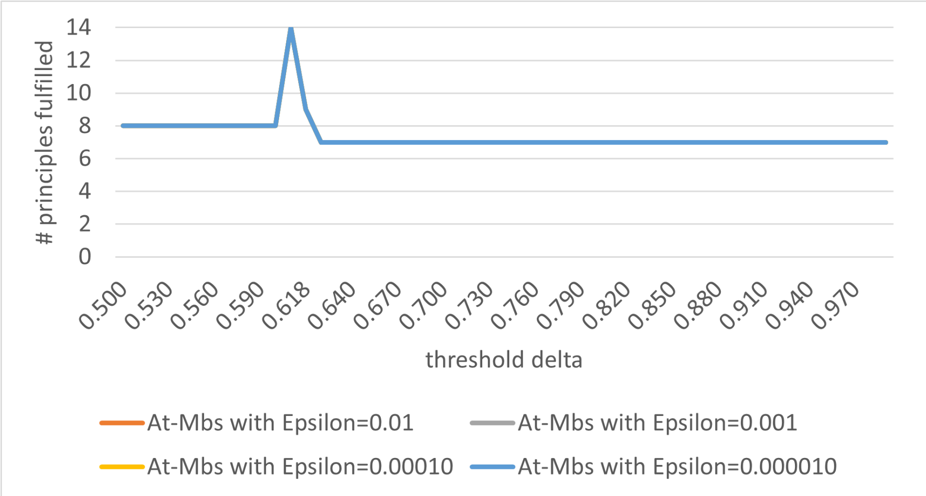

Figure 33: Non-detailed threshold evaluation using *At-Mbs*. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

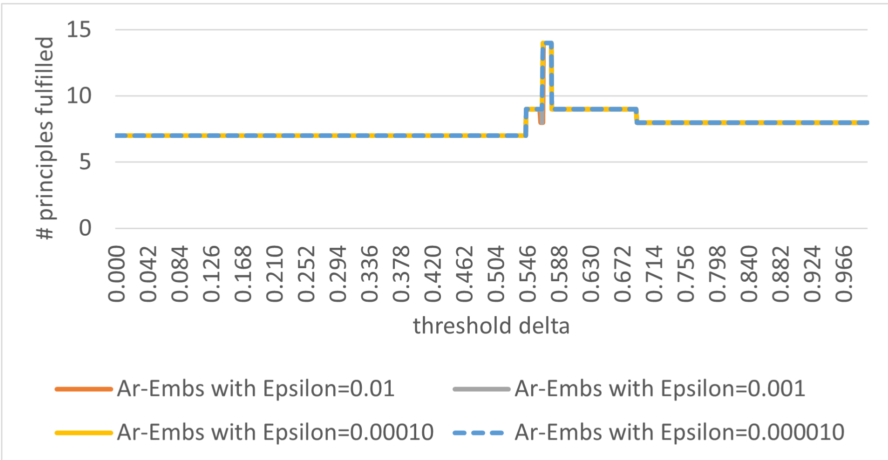

Figure 34: Non-detailed threshold evaluation using *Ar-Embs*. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

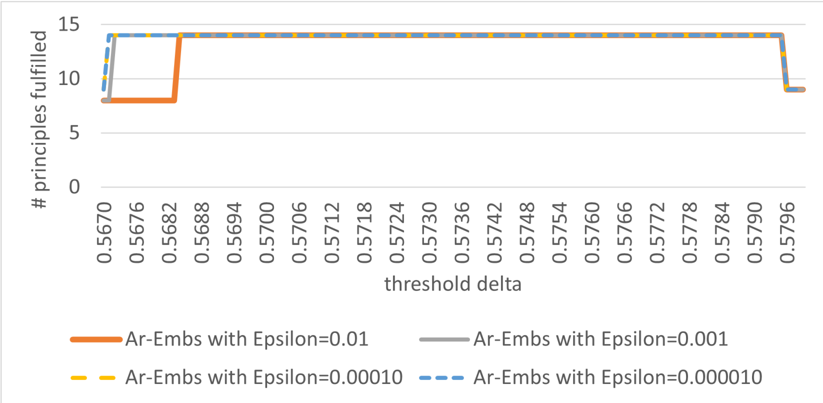

Figure 35: Detailed threshold evaluation using *Ar-Embs*. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

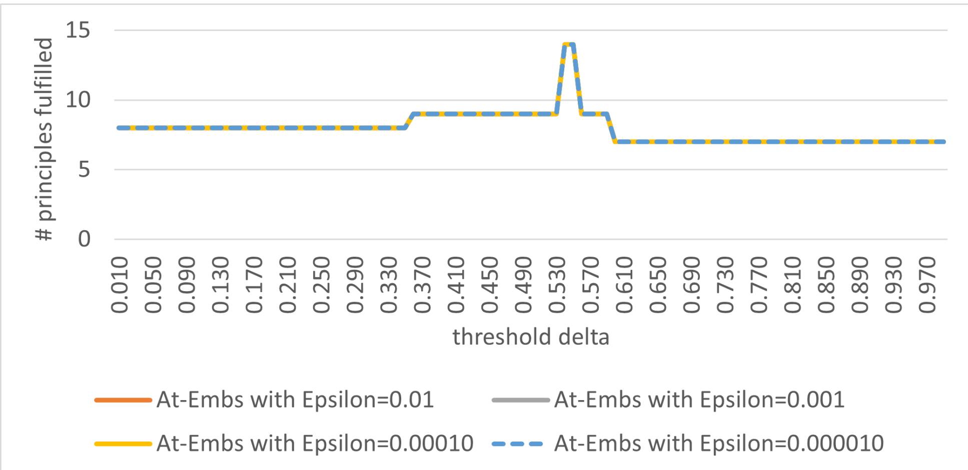

Figure 36: Non-detailed threshold evaluation using *At-Embs*. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

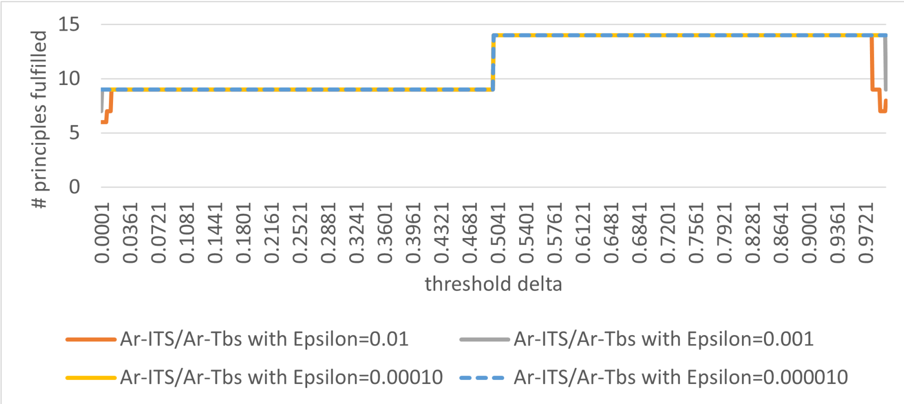

Figure 37: Non-detailed threshold evaluation using *Ar-ITS* resp. *Ar-Tbs*. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

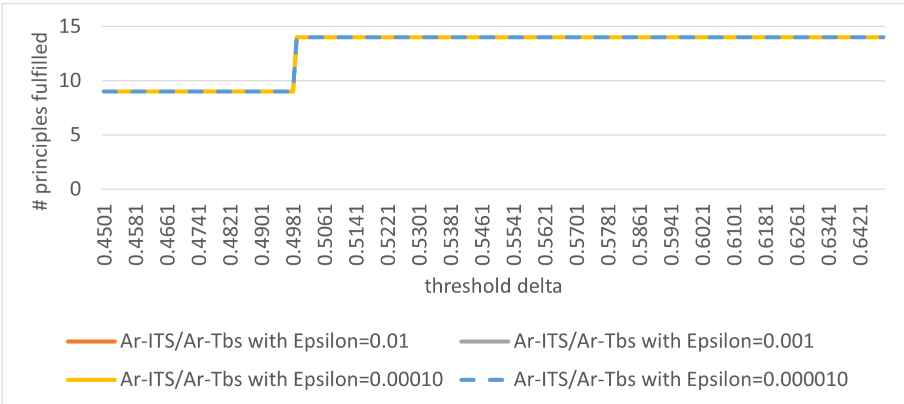

Figure 38: Detailed threshold evaluation using *Ar-ITS* resp. *Ar-Tbs*. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

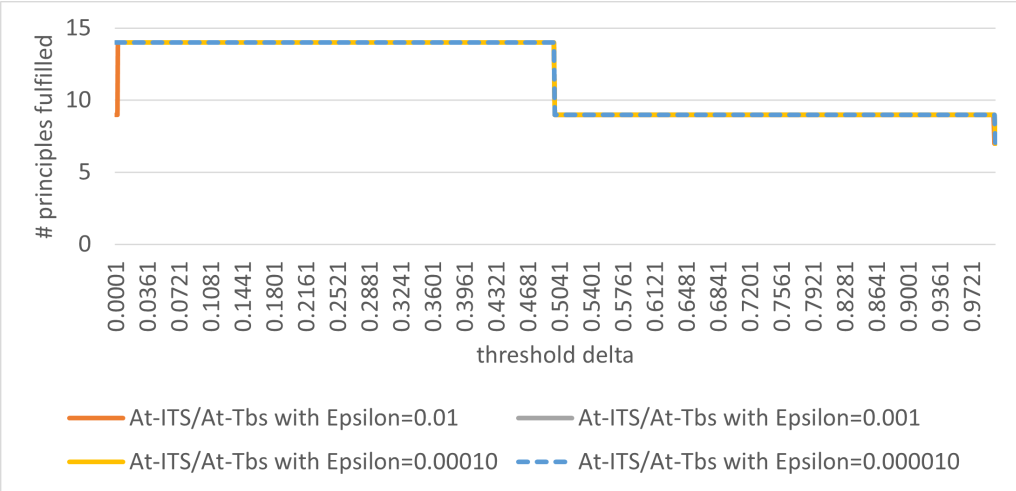

Figure 39: Non-detailed threshold evaluation using *At-Tbs/At-ITS*. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

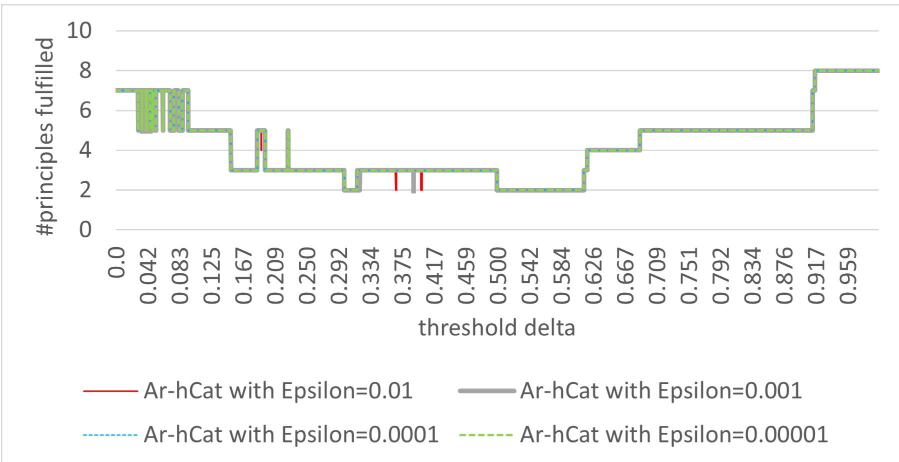

Figure 40: Non-detailed threshold evaluation using *Ar-hCat*. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

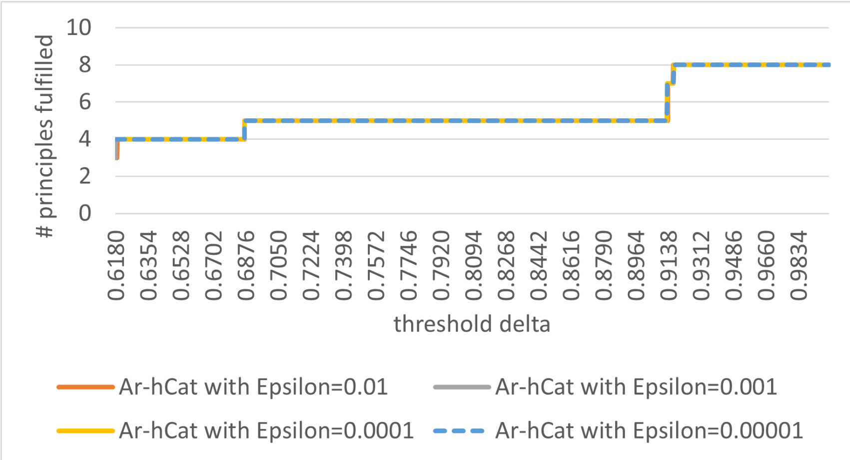

Figure 41: Detailed threshold evaluation using Ar-hCat. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

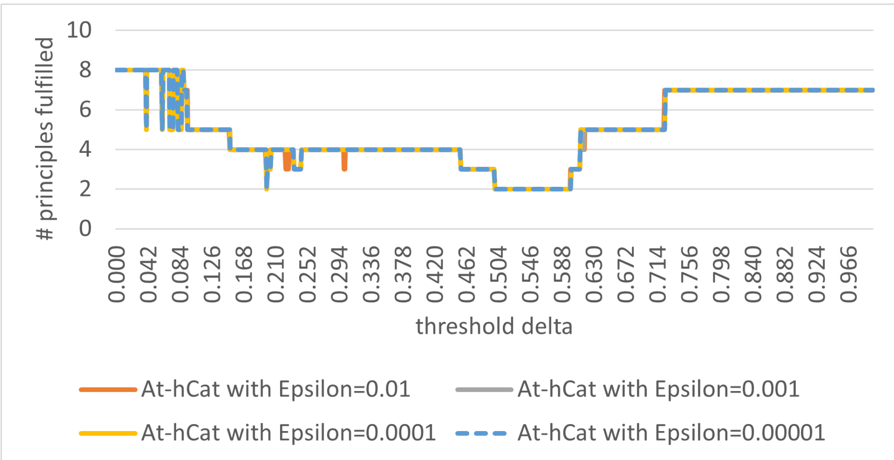

Figure 42: Non-detailed threshold evaluation using At-hCat. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

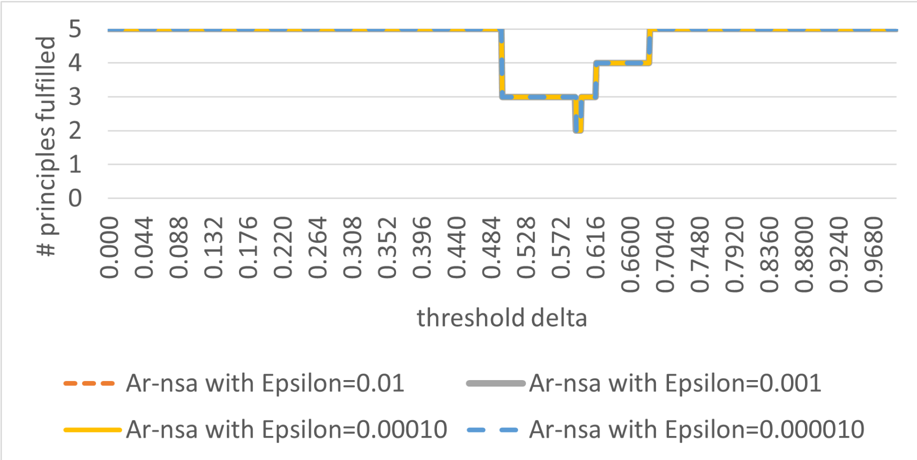

Figure 43: Non-detailed threshold evaluation using *Ar-nsa*. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

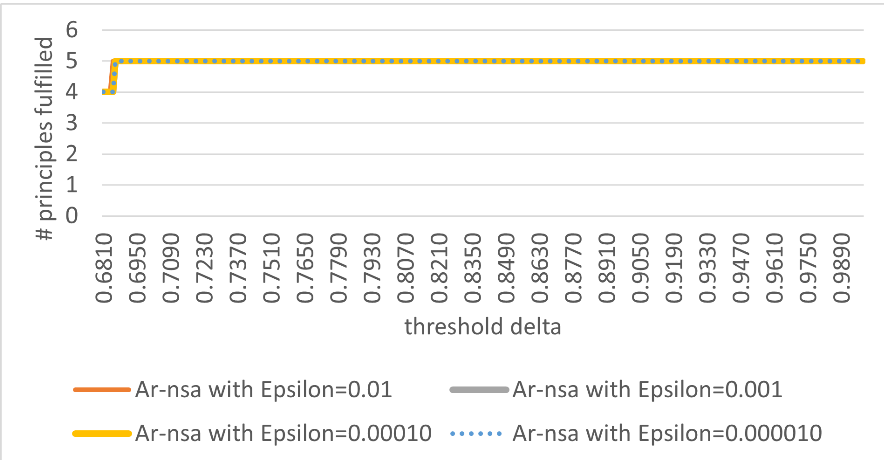

Figure 44: Detailed threshold evaluation using Ar-nsa. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

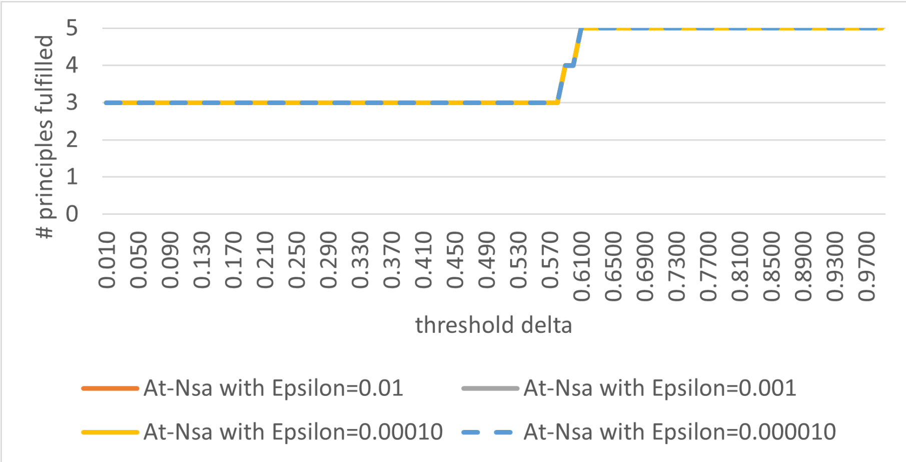

Figure 45: Non-detailed threshold evaluation using At-nsa. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

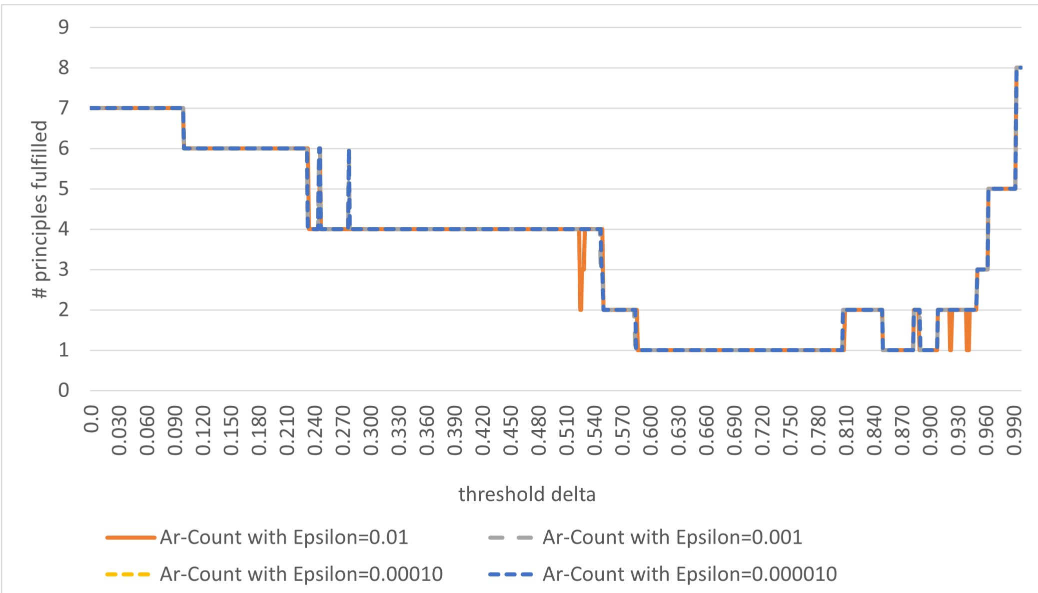

Figure 46: Non-detailed threshold evaluation using *Ar-Count*

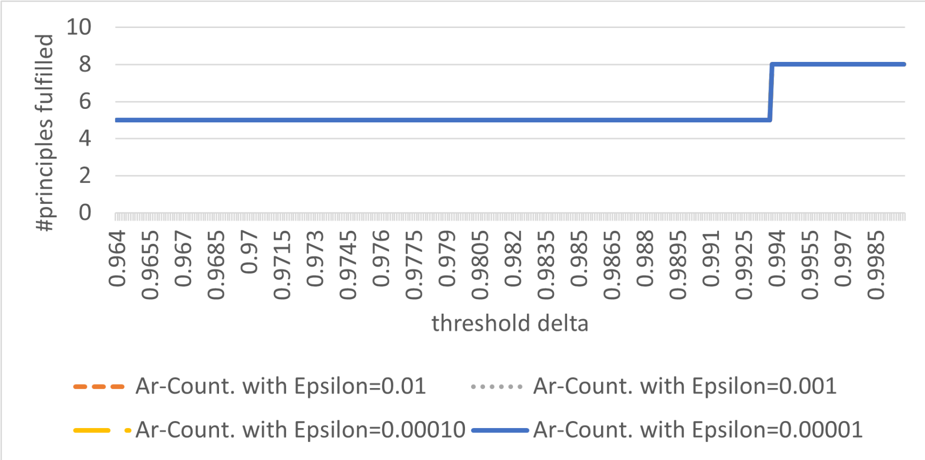

Figure 47: Detailed threshold evaluation using *Ar-Count*. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

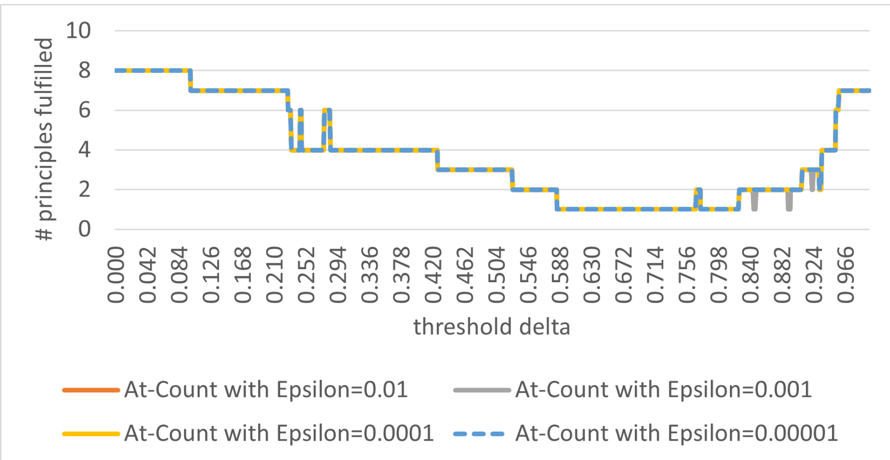

Figure 48: Non-detailed threshold evaluation using *At-Count*. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.

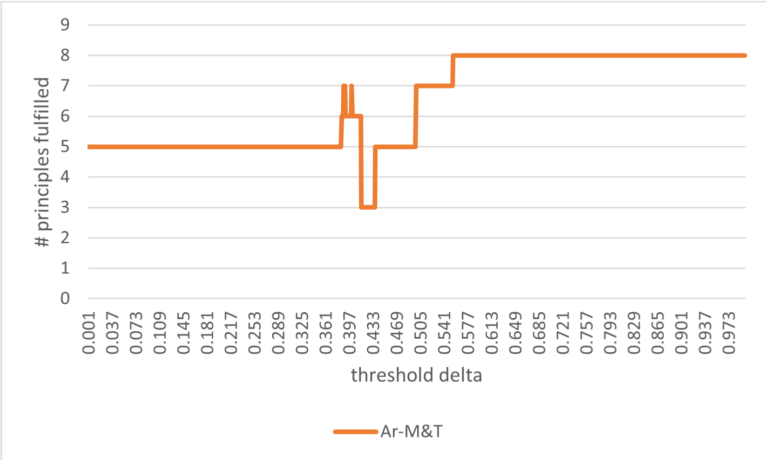

Figure 49: Non-detailed threshold evaluation using *Ar-M&T*

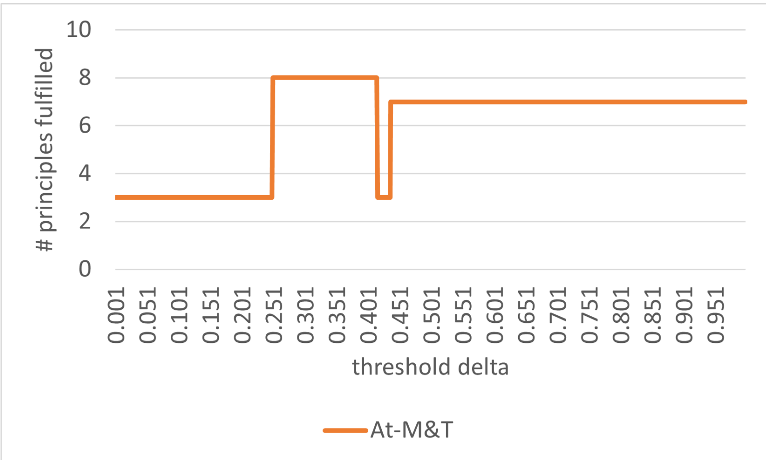

Figure 50: Non-detailed threshold evaluation using *At-M&T*. The term *principles fulfilled* refers to the principles not disproven in the experimental evaluation.